Permutations & Combinations

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ADAPTED FROM SUSANA EPP:
DISCRETE MATHEMATICS
WITH APPLICATIONS

Lecture Overview

- 2 Necessary Tools to Understand Permutations and Combinations
  - Multiplication Principle
  - Addition Principle
- Permutations
  - R-Permutation
- Combinations
  - R-Combinations

Multiplication Principle
&
Addition Principle:

THE 2 TOOLS WE NEED TO UNDERSTAND

PERMUTATIONS & COMBINATION

Illustration of Multiplication Principle

CAFÉ EXAMPLE

A) How many possible meals consisting of 1 MC & 1 Bev?

Illustration of Multiplication Principle

B) How many possible meals consisting of 1 App & 1 MC & 1 Bev?

C) How many possible meals consisting of 1 App & 1 MC & 1 optional Bev?

Definition of Multiplication Principle

What we have been applying is known as the **multiplication principle,** whereby:

We **multiply** together the **number of ways** of doing **each step** when one activity is constructed in **successive steps. (think AND)**

Illustration of Addition Principle

TEXTBOOKS EXAMPLE

5 distinct CS books

3 distinct Math books

2 distinct Art books

How many ways can we select 2 books from different subjects from among the books above?

Definition of Addition Principle

What we have been applying is known as the **addition principle,** whereby:

We **add** together the **number of elements** in **each subset** (option) when the elements being counted can be decomposed (separated) into **disjoint sets. (think OR)**

Example 3

How many ways can 3 people sit in a row, next to each other?

Example 4

There is a 6-person committee consisting of _ _ _ _ _ _

There are 3 positions to be filled (P, VP, T) [one person can hold only one office]

A) How many ways can we select 3 officers from the 6 people?

B) How many ways can this be done if either ___________________ or ___________________ must be president (P)?

Permutation

Recall Example 3

What we have effectively illustrated is **permutation** (ordering)

P(n) = n! [the permutation of n objects (in a row) is n factorial]

In how many ways can 3 people sit in a row?

P(3) = 3!

How about if the 3 people were sitting a circle, then in how ways can they sit in a row?

r-Permutation

Recall Example 4

r-permutation: sometimes, we want to consider an ordering of r elements selected from n available elements

$P(n, r) = n!/(n-r)!$

Example 5:

A) There are 8 friends at the movies. In how many ways can they sit together in a row?

B) One of the friends is a medical doctor & needs to sit in an aisle seat (assume each row only has 8 seats and thus, there are 2 aisle seats). In how many ways can the friends now sit? [This can be done using both MP and AP]

C) The 8 friends consist of 4 couples, who naturally want to sit next to each other. But for some odd reason, the older of each couple needs to sit on the left-side of the younger one. In how many ways can the 8 friends now sit next to each other? [MD no longer on call]

D) The 8 friends consist of 4 couples, who naturally want to sit next to each other. In how many ways can the 8 friends now sit next to each other? [No odd conditions exist here. MD no longer on call]

The Photoshoot (part 1)

In how many ways can 7 distinct Martians and 5 distinct Earthlings **stand in line** for the photo of the year – but **no 2 Earthlings can stand together**.

Step 1 (line up the Martians)

Step 2 (line up the Earthlings)

Step 3

The Photoshoot (part 2)

In how many ways can 7 distinct Martians and 5 distinct Earthlings **stand in a circle** for the photo of the year – but **no 2 Earthlings can stand together**.

Step 1 (circle up the Martians)

Step 2 (circle up the Earthlings)

Step 3

Combination

Combination

One or more elements selected from a set without regard to order.

Consider 10 students who have come into my office, at the same time, to work on a project – in how many ways can they do so (at the same time)

You will see that only r-combination is of use to us ...

r-Combination

r-Combination

$C(n, r) = P(n, r)/ r!$ $\qquad$ $n >= 1$

Similar to r-permutation but will have _________ (more or less?) outcomes as there is no emphasis on order

Example 6

In how many ways can we select a committee (or group or team) of 3 people from a group of 10 students?

Example 7

There are 12 employees in a company

A) In how many ways can 5 of the employees be chosen to work on a project together? i.e. how many 5-person teams can be formed?

[Do you see why this is combination and not permutation?]

B) There are 2 employees (A & B) who must always work together. Now, in how many ways can 5 of the employees be chosen to work on a project together? i.e. how many 5-person teams can be formed given this condition?

C part 1) There are 2 employees (C & D) who refuse to work together. Now, in how many ways can 5 of the employees be chosen to work on a project together? (previous condition with A & B no longer applies)

[Addition method]

C part 2) There are 2 employees (C & D) who refuse to work together. Now, in how many ways can 5 of the employees be chosen to work on a project together?

[Difference method]

Example 8

There are 12 students applying for an internship: 5 from Maryland and 7 from Wisconsin

A) In how many ways can we choose 5 students so that 3 are from Maryland and 2 from Wisconsin?

B) In how many ways can we choose 5 students so that at least 1 is from Maryland?

[Addition rule or Difference rule?]

C) In how many ways can we choose 5 students so that at most 1 is from Maryland?

(Addition Principle or the Difference Method (within the AP)?)

Example 9

How many strings can be formed using the following letters:

M I S S I S S I P P I

Homework # 7