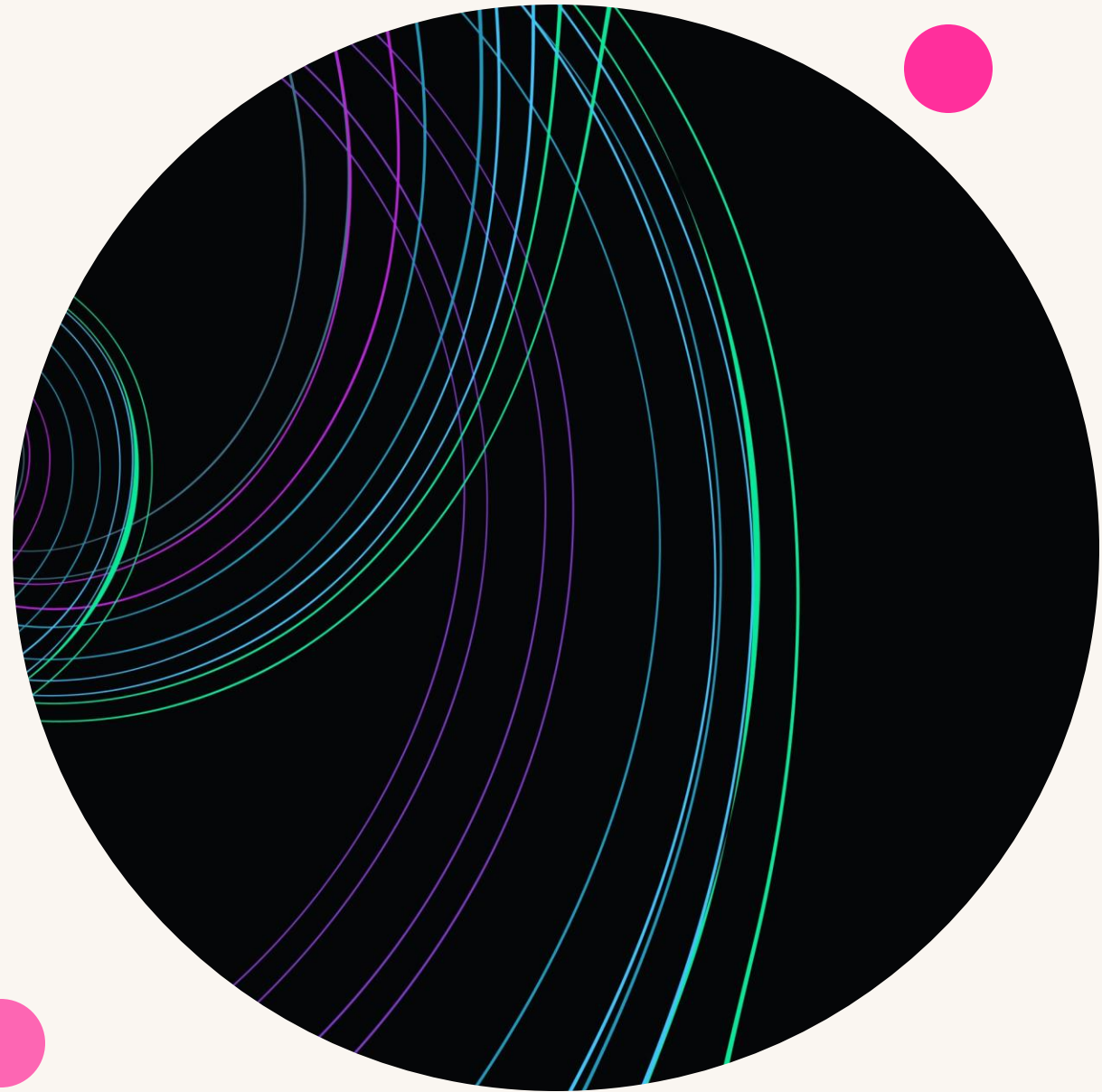


Pumping Lemma

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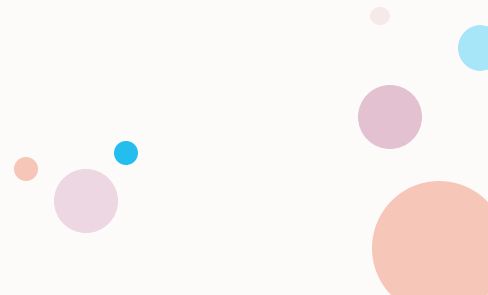
Adapted from "Introduction to Computer Theory" by Daniel Cohen





Introduction

- A regular language can be defined by an FA/ FSM
- Is there a way for us to effectively show that a language is **not regular**?



Pumping Lemma

- The **pumping lemma** is useful for **disproving the regularity** of a specific language.
- The Pumping lemma basically states that all sufficiently long strings (S) in a **regular language** (A) may be "pumped" – that is,
 - have a middle section of the string (y^n) repeated in an arbitrary number of times (*I often use 2 as my arbitrary #*) – to produce a new string (S^1) that is also a part of the language.

Pumping Lemma

- If A is a regular language, then A can be pumped.
- A has a pumping length (some constant, P) such that any string S (where $|S| \geq P$) may be divided into 3 parts (x , y , & z)
- $S = x y z$ (3 substrings, x , y , z , with y being non-empty, such that the strings constructed by repeating y , 0 or more times, are still in A).
 - $|xy| \leq P$ (i.e. the length of x followed by y must be less or equal to the pumping length, P , thus, imposing a limit on how S may be split)

In addition:

- Any string, S^1 , constructed from $x y^n z$ (where $n \geq 0$) is also an element of A
- y cannot be λ (i.e. y must be non-empty)

Proof by Contradiction

- Assume the language A is regular
 - A has to have a pumping length: P
 - (Since A is regular) Any strings, S , in language A , that is longer than P , can be pumped
 - Find $S \mid S \geq P$
 - Divide S into xyz , where $|xy| \leq P$
 - Consider the ways that S can be divided into $x y z$

 - Consider S^1 , which is also an element of language A
- (CONTRADICTION: Show that the necessary conditions cannot be satisfied at the same time)
- Show $S^1 = x y^n z$ (where $n \geq 0$) cannot be "pumped" [remember we need to show only ONE counterexample to disprove "all"]
 - Conclude that S^1 cannot be pumped, because it is not a regular language. S^1 is an element of language A - thus A is also not a regular language

Pumping Lemma

Example 1: Language $A = \{a^n b^n \mid n \geq 0\}$

Before we get started, consider:

- This language has any number of a's followed by the same number of b's
- Consider that we need to keep a count of a's to determine how many b's - but FA cannot keep count of anything. FA cannot be used to define this A. (Note: Regular languages can be defined by FA's)
- Keep the following approach in mind as we go through the examples:
 1. For the Proof by Contradiction, suppose A is a regular language
 2. We need a pumping length, P | any string S, in language A, is greater than or equal to P ($S \geq P$)
 3. Let's use pumping length, $P = 7$ and let's assign string $S = a^p b^p$
 4. Consider the ways that S can be divided into xyz, where $|xy| \leq P$
 5. Now consider $S' = xy^n z$, where $n \geq 0$
 6. Show that S' cannot be "pumped." [we need to show only ONE counterexample to disprove "all"]
 7. Conclude: String S' is in language A. If S' cannot be "pumped" then A cannot be "pumped," and therefore, A is not a regular language

Pumping Lemma

Example 1: Use the pumping Lemma to prove, language $A = \{a^n b^n \mid n \geq 0\}$ is not regular
[This language has any number of a's followed by the same number of b's]

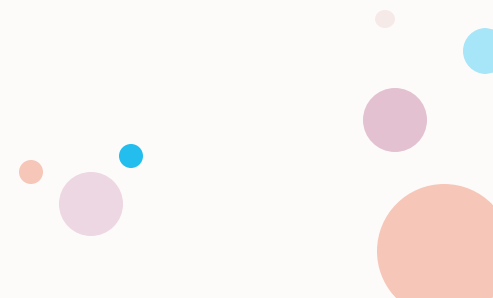
Proof by Contradiction

- Suppose A is a regular language
 - S is an element of A (i.e. S is a word in the language A)
 - Let pumping length, P be 7
 - Let's assign $n = 6$, so string $S = a^6 b^6$
 - Consider the ways that S can be divided into xyz , where $|xy| \leq P$
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- Now consider $S' = xy^m z$, where $m = 2$
 - Contradiction: Show that S' cannot be "pumped."
 - Conclude: String S' is in language A . If S' cannot be "pumped" then A cannot be "pumped," and therefore, A is not a regular language

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Pumping Lemma

Example 1: Continued



Pumping Lemma

Example 2: Use the pumping Lemma to prove, language $A = \{y y \mid y \in (0^n 1)\}$ is not regular
[This language has any number of 0's followed by 1, and then repeated once more (y y)]

Proof by Contradiction

- Suppose A is a regular language
- S is an element of A (i.e. S is a word in the language A)
- Let pumping length, P be 7
- Let's assign $n = 7$, so string $S = 0^7 1$
- Consider the ways that S can be divided into xyz , where $|xy| \leq P$
- Now consider $S' = xy^mz$, where $m = 2$
- Show that S' cannot be "pumped." (in a regular manner) [we need to show only ONE counterexample to disprove "all"]
- Conclude: String S' is in language A . If S' cannot be "pumped" then A cannot be "pumped," and therefore, A is not a regular language

Pumping Lemma

Example 2: Continued

(Consider the ways that S' can be divided into xy^nz and show that S' cannot be "pumped" (let's assign $n=2$).

String S' is in language A . If S' cannot be "pumped" then A cannot be "pumped," and therefore, A is not a regular language.)