Pumping Lemma

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Introduction

- A regular language can be defined by an FA/ FSM
- Is there a way for us to effectively show that a language is **not regular**?

Pumping Lemma

- The pumping lemma is useful for disproving the regularity of a specific language.

- The Pumping lemma basically states that all sufficiently long strings *(S)* in a **regular language** *(A)* may be "pumped" – that is,
  - have a middle section of the string *(y^n)* repeated in an arbitrary number of times *(I often use 2 as my arbitrary #)* – to produce a new string *(S^1)* that is also a part of the language.

Pumping Lemma

- If A is a regular language, then A can be pumped.

- A has a pumping length (some constant, P) such that any string S (where S>=P) may be divided into 3 parts (x, y, & z)

- S = x y z (3 substrings, x, y, z, with y being non-empty, such that the strings constructed by repeating y, 0 or more times, are still in A).
  - |xy| <=P (i.e. the length of x followed by y must be less or equal to the pumping length, P, thus, imposing a limit on how S may be spilt)

In addition:

- Any string, S^1, constructed from x y^nz (where n>=0) is also an element of A
- y cannot be lambda (i.e. y must be non-empty)

Proof by Contradiction

- Assume the language A is regular
- A has to have a pumping length: P
- (Since A is regular) Any strings, S, in language A, that is longer than P, can be pumped
- Find S | S>=P
- Divide S into xyz, where |xy|<=P
- Consider the ways that S can be divided into x y z

- Consider S^1, which is also an element of language A

(CONTRADICTION: Show that the necessary conditions cannot be satisfied at the same time)

- Show S' = x y^nz (where n>=0) cannot be "pumped" [remember we need to show only ONE counterexample to disprove "all"]
- Conclude that S' cannot be pumped, because it is not a regular language. S' is an element of language A - thus A is also not a regular language

Pumping Lemma

Example 1: Language A = $\{a^n b^n \mid n>=0\}$

<u>Before we get started, consider:</u>

- This language has any number of a's followed by the <u>same</u> number of b's
- Consider that we need to keep a count of a's to determine how many b's - but FA cannot keep count of anything. FA cannot be used to define this A. (Note: Regular languages can be defined by FA's)

- Keep the following approach in mind as we go through the examples:

1. For the Proof by Contradiction, suppose A is a regular language
2. We need a pumping length, P | any string S, in language A, is greater than or equal to P (S>=P)
3. Let's use pumping length, P = 7 and let's assign string S = $a^p b^p$
4. Consider the ways that S can be divided into xyz, where |xy|<=P
5. Now consider S' = $xy^n z$, where n >=0
6. Show that S' cannot be "pumped." [we need to show only ONE counterexample to disprove "all"]
7. Conclude: String S' is in language A. If S' cannot be "pumped" then A cannot be "pumped," and therefore, A is not a regular language

Pumping Lemma

Example 1: Use the pumping Lemma to prove, language $A = \{a^n b^n \mid n >= 0\}$ is not regular

[This language has any number of a's followed by the same number of b's]

Proof by Contradiction

- Suppose A is a regular language
- S is an element of A (i.e. S is a word in the language A)
- Let pumping length, P be 7
- Let's assign n = 6, so string $S = a^6 b^6$
- Consider the ways that S can be divided into xyz, where $|xy| <= P$

- Now consider S' = $xy^m z$, where m =2
- Contradiction: Show that S' cannot be "pumped."
- Conclude: String S' is in language A. If S' cannot be "pumped" then A cannot be "pumped," and therefore, A is not a regular language

Pumping Lemma

Example 1: Continued

Pumping Lemma

Example 2: Use the pumping Lemma to prove, language A = {y y | y ϵ (0^n 1)} is not regular

[This language has any number of 0's followed by 1, and then repeated once more (y y)]

Proof by Contradiction

- Suppose A is a regular language
- S is an element of A (i.e. S is a word in the language A)
- Let pumping length, P be 7
- Let's assign n = 7, so string S = 0^7 1
- Consider the ways that S can be divided into xyz, where|xy|<=P

- Now consider S' = xy^mz, where m = 2
- Show that S' cannot be "pumped." (in a regular manner) [we need to show only ONE counterexample to disprove "all"]
- Conclude: String S' is in language A. If S' cannot be "pumped" then A cannot be "pumped," and therefore, A is not a regular language

Pumping Lemma

Example 2: Continued

(Consider the ways that S' can be divided into xy^nz and show that S' cannot be "pumped" (let's assign n=2).

String S' is in language A. If S' cannot be "pumped" then A cannot be "pumped," and therefore, A is not a regular language.)