DURABILITY OF PROFESSIONAL AND SOCIOMATHEMATICAL NORMS FOSTERED IN A MATHEMATICS METHODS COURSE

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This study investigated the extent to which seven professional and sociomathematical norms intentionally fostered in a mathematics methods course through the use of a video case professional development curriculum re-emerged in a later course with a different cohort. All seven norms were evident, to varying extents, in the written analysis and group discussion of 11 prospective teachers who engaged in a video case analysis similar to those they had participated in during their first methods course one to four semesters earlier. This paper discusses the norms, evidence, and counterevidence for their re-emergence, and implications for teacher preparation.

Objectives

A key intended outcome of our mathematics teacher education program is for prospective teachers to experience self-sustaining generative change, defined by Franke, Carpenter, Fennema, Ansell, and Behrend (1998) to involve “teachers changing in ways that provide a basis for continued growth and problem solving” (p. 67). This paper analyzes the durability of one component of our efforts to achieve this outcome—the development of professional and sociomathematical norms embedded in the Learning and Teaching Linear Functions (LTLF) video case professional development curriculum (Seago, Mumme, & Branca, 2004).

Specifically, we address the extent to which professional and sociomathematical norms intentionally fostered in a mathematics methods course re-emerge in a similar context later in the program with a different cohort of prospective teachers.

Perspectives

Since the identification of sociomathematical norms as critical contributors to school mathematics learning (Yackel & Cobb, 1996), a growing body of research has investigated the subtle power of these norms to support the development of mathematical learners (e.g. Kazemi & Stipek, 2001; McClain & Cobb, 2001). More recently, researchers have turned their attention to the role of such norms in supporting teachers’ learning during professional development (e.g. Elliott et al., in preparation; Grant, Lo, & Flowers, 2007).

Recognizing that sociomathematical norms have the potential to support teachers’ learning, Seago, Mumme, and Branca (2004) incorporated the development of such norms, as well as a set of professional norms, into the LTLF materials that we adapted for use in our mathematics methods course. These professional and sociomathematical norms are listed in Column 1 of Table 1 and form the basis of our study. We see these norms as important to preparing teachers in three ways: (1) supporting the development of their own mathematical understanding;
(2) learning to view and analyze classroom practice in productive ways; and (3) thinking about what norms should be developed in mathematics classrooms with students.

One of the challenges of investigating norms is the difficulty of determining normative behavior from a snapshot of practice. For example, in one class session, there may not be time for each participant to present a solution to a mathematical task, but if most who do include a mathematical argument, a reasonable inference is that mathematical argumentation is a norm for that class. On the other hand, if few include a mathematical argument, it is safe to conclude that it is not a norm. Another way to infer the presence of a norm is when the norm is not exhibited and this is recognized and corrected by other members of the group.

We approach both the development of the program and our research from a situated perspective (e.g. Borko et al., 2000). That is, we generate learning situations that are similar to those in which we intend prospective teachers to use the learning in their future teaching, and we study the way in which they interact in these situations. We also follow Cobb, Stephan, McClain, and Gravemeijer (2001) in our interest in coordinating the social and psychological perspectives. For the study reported here, this means that we have concerned ourselves with evidence of professional and sociomathematical norms in both group interactions (social perspective) and in the prospective teachers’ individual written work (psychological perspective).

**Modes of Inquiry**

The participants in the study were 11 prospective mathematics teachers (PTs) enrolled in their final mathematics methods course who had used the LTLF video case curriculum in their first mathematics methods course one to four semesters earlier. During one 80-minute class session, these PTs were engaged in a video case discussion similar to those they had participated in during their first methods course. Prior to the session, the PTs solved the mathematics problem (see Figure 1) individually (Data Source 1) and predicted possible student correct (DS 2) and incorrect (DS 3) thinking. The session began with a group discussion about the mathematics. Next, the PTs watched a video of middle school students discussing their thinking about the same problem and responded in writing to questions about what they noticed (DS 4), student thinking (DS 5 and DS 6), and teacher actions (DS 7) in the video, and then engaged in a group discussion about these ideas. At the end of the session, the PTs reflected in writing about what they learned from the discussion (DS 8). In addition to these written data sources, the group discussions of the mathematics and the video were videotaped and transcribed (T).

![Figure 1. Counting Cubes problem solved by the PTs and the students in the video. The problem and the video are from the Turning to the Evidence project (see Seago & Goldsmith, 2005).](image-url)

The first two authors facilitated the session. Both had taught the first methods course, but half the participants had taken it from other instructors. The facilitators made a point of not

introducing professional or sociomathematical norms in order to see if any of the norms established in the first methods course would spontaneously re-emerge.

In order to analyze the data for evidence of the seven professional and sociomathematical norms (see Column 1, Table 1), transcripts of the mathematics and video discussion and all written work were coded independently by at least two researchers for examples and counterexamples of each norm. Any differences were resolved through refining the code definitions. The analysis was completed using multiple charts that cross-referenced evidence and counterevidence of each norm by PT. These charts were then collapsed into the summary chart shown in Table 1.

Results and Discussion

Overview

Table 1 summarizes the professional and sociomathematical norms exhibited by each PT, as well as the identifier of the data source in which each norm was exhibited [1-8, T]. “C” indicates that a counterexample of the norm was identified. For example, Abby’s “TC” for talking with respect yet engaging in critical analysis indicates that she was critical, but not respectful of the teacher in the video in at least one instance in the class discussion.

The bottom two rows list the number and percent of speaking turns during the discussion. It is worth noting that Hana spoke only during the mathematics portion of the discussion when she presented a possible way that students might think about the problem, Abby spoke at the end of the video discussion after being encouraged to do so by the facilitator, and Ruth did not speak at all, despite a direct invitation.

Table 1.

<p>| Professional and Sociomathematical Norms Exhibited by Prospective Teachers |
|---|---|---|---|---|---|---|---|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>Norm</th>
<th>Abby</th>
<th>Evan</th>
<th>Hana</th>
<th>Iris</th>
<th>Jim</th>
<th>Ken</th>
<th>Leah</th>
<th>Lily</th>
<th>Lew</th>
<th>Roxy</th>
<th>Ruth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Listening to and making sense of or building on others’ ideas</td>
<td>4, 6</td>
<td>T, 6, 8</td>
<td>7</td>
<td>8</td>
<td>T, 8</td>
<td>T, 1, 4, 5, 8</td>
<td>T, 4, 8</td>
<td>4</td>
<td>T, 8</td>
<td>T, 4</td>
<td></td>
</tr>
<tr>
<td>Adopting a tentative stance toward practice – wondering vs. certainty</td>
<td>8</td>
<td>T</td>
<td>8</td>
<td>T</td>
<td>T, TC</td>
<td>T, 8</td>
<td>T, TC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Backing up claims with evidence and providing reasoning</td>
<td>T, 7</td>
<td>T</td>
<td>T, 7</td>
<td>T</td>
<td>T</td>
<td>7</td>
<td>4, 6, 8</td>
<td>T, 4, 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Talking with respect yet engaging in critical analysis of teachers and students portrayed on the video</td>
<td>TC</td>
<td>T</td>
<td>T</td>
<td>T, TC, 7, 8</td>
<td>T</td>
<td>T, 4</td>
<td>T, 6</td>
<td>T</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Naming, labeling, distinguishing, and comparing mathematical ideas</td>
<td>4, 5, 8</td>
<td>T, 4, 5, 6, 7</td>
<td>4, 5, 7</td>
<td>T, 4, 5</td>
<td>T, 4, 5, 8</td>
<td>T</td>
<td>T, 4</td>
<td>T, 4, 5</td>
<td>4, 5, 7, 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Using mathematical explanations that consist of a mathematical argument, not simply a procedural description or summary</td>
<td>T, 1, 4, 6</td>
<td>T, TC, 1</td>
<td>1</td>
<td>T, 1, 5</td>
<td>T, TC, 1, 6</td>
<td>T, 1, 6</td>
<td>T, 1, 6</td>
<td>T, 1, 4, 6</td>
<td>T, 1, 5</td>
<td>T, 1</td>
<td></td>
</tr>
<tr>
<td>Raising questions that are related to the mathematics and push on understanding of another’s mathematical reasoning</td>
<td>7, 8</td>
<td>4</td>
<td>7, 8</td>
<td>T, 7</td>
<td>T, 4, 7, 8</td>
<td>T</td>
<td>4</td>
<td>T, 7</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Speaking Turns</td>
<td>3</td>
<td>21</td>
<td>9</td>
<td>12</td>
<td>22</td>
<td>18</td>
<td>15</td>
<td>6</td>
<td>17</td>
<td>19</td>
<td>0</td>
</tr>
<tr>
<td>% Participant Speaking Turns</td>
<td>0%</td>
<td>15%</td>
<td>6%</td>
<td>8%</td>
<td>15%</td>
<td>13%</td>
<td>11%</td>
<td>4%</td>
<td>12%</td>
<td>13%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Note. Professional and sociomathematical norms are from Seago, Mumme, and Branca (2004).
As can be seen in Table 1, 3 of the 11 PTs (Iris, Ken, and Lew) exhibited each of the seven norms in the discussion and/or their written work with no counterexamples. An additional 2 PTs (Jim and Leah) exhibited each of the seven norms, but Leah also had a counterexample for the norm of adopting a tentative stance toward practice, while Jim had counterexamples of both talking with respect, yet engaging in critical analysis and using a mathematical argument. Three of the remaining PTs exhibited six norms overall (Abby, Evan, and Roxy), 1 exhibited five norms (Lily), and 2 PTs exhibited three (Hana and Ruth).

In the whole-group discussion, 7 of the 11 PTs (Evan, Iris, Jim, Ken, Leah, Lew, and Roxy) exhibited at least five of the norms. The other 4 PTs exhibited zero (Hana and Ruth), two (Abby), and three (Lily) of the norms, but they all participated in the discussion in a very limited way. As might be expected, there appears to be a general correspondence between the percent of speaking turns during the discussion and the number of norms exhibited, with the largest number of norms exhibited by those PTs who participated the most. Iris is an exception to this, however, as she had only 8% of the speaking turns, yet exhibited five of the seven norms.

To better understand evidence and counterevidence of the norms during the discussion, consider an exchange that occurred at the end of the session and contained Abby’s entire contribution to the video component of the discussion:

F2: There’s a few people that we haven’t heard from yet. Wondering if any of you who’ve not talked a lot have something that you want to add to [previous comment]. Like Abby.

Abby: I didn’t like the teacher. I didn’t like that he didn’t ask about the picture. I felt like he was feeding them the answers. I hated line 85. So I have nothing good to say about the teacher, which,

F2: Can you say a little bit more about what it was that—

Abby: I feel like he should have tried to make more connections about the picture, because, like we were talking about, group 2 didn’t really know what the minus 4 was; it just worked for the numbers, and I think he could have tried to pull more out of them. Same with what Iris was saying about group 3. We know nothing about their method. He just kind of let them put that up there, and then used it to make the, the number connection about the expressions, the algebraic stuff. Like he could have made more connections with the visual, uh, aspect of it. And then I just didn’t feel, like he just talked [brief pause] at the end, in line 85, like he could have let them explain that more.

F2: Okay. You were storing up a lot. [laughter] So there’s several things to respond to that—

Jim: I felt the same way, because in line 26, he, you know, “How many cubes would be in a [seventh] building?” And Cassie says “31,” and how, you know, he asked how they got that. They’ve already made it apparent that they know how to substitute, because, before that, Cassie says that they first thought it was 5n + 1, but they found out that it didn’t work for the first one. And so, he just asks a redundant question, and, by asking them what the seventh one was, all they have to do is plug it in. But he never asks why it’s 5n – 4. He just, they give him the answer, he asks what the seventh one is, they give the right answer and it’s over. But never why, where did the minus 4 come from? Again, never relates it to the picture.

Iris: I guess it kind of depends on what his teacher goal was for the day. [laughter] [T539-564]

Both Abby and Jim were coded as exhibiting a counterexample (TC) for the critical yet respectful norm because they did not talk about the teacher’s practice in a respectful way during this exchange. Iris, on the other hand, shifted the conversation back toward showing respect towards the teacher by raising the question of whether his actions might have been appropriate.
for his goals for the day. Although it is difficult to see emotions in a transcript, the points at which the laughter occurred support the facilitators’ sense that Abby’s and Jim’s comments made the group uncomfortable, and thus were not normative. It seems that the laughter served the role of diffusing tension caused by a violation of a group norm and the action taken by Iris to re-establish it. Also interesting to note is that both Abby and Jim used evidence to back up their claims, demonstrated that they had been listening to the prior conversation, and, in fact, raised valid concerns about the teacher’s actions. Thus, even though they were not respectful, they were critical in a potentially productive way. In our experience, however, the respectful component is important to developing an atmosphere where teachers feel comfortable talking about their teaching and, in so doing, are able to identify and act on specific opportunities to improve their practice. If the group had not self-corrected, the facilitator would have used this exchange to point out the importance of being critical in a respectful way and to remind the PTs that their analysis should focus on the instance of teaching practice, not the teacher himself.

In the written work, all 11 PTs exhibited at least three norms, with 5 exhibiting five or more of the seven norms. The PTs demonstrated the norms in two ways: by participating in the norms themselves and by making a statement that indicated they recognized that the norm was important in the classroom. For example, Lew exhibited the norm of using a mathematical argument both ways. First, he made a mathematical argument to justify a mathematical expression: “A better more visual formula would be 5(n – 1) + 1, since I have five ‘arms’ that are (n – 1) long and one single ‘central’ block” [DS 1]. Second, he recognized the importance of the norm in the classroom when he reflected that the students in the video “were very thorough and were very aware of what everything stood for in their solution” [DS 5]. Each PT demonstrated between two and five norms in their own analyses and reflections, and made statements indicating they recognized the importance of between one and five norms. Looked at in another way, over half of the PTs made statements alluding to the importance of the norms that could be considered most relevant to their future classrooms: listening to and making sense of others’ ideas; backing up claims with evidence; naming, labeling, distinguishing, and comparing mathematical ideas; using mathematical arguments; and raising questions that push on others’ understanding. This is important as these future teachers will not likely work to establish these norms in their own classrooms unless they are explicitly aware of the norms and recognize them as important to developing mathematical understanding.

In sum, all seven norms fostered in the first mathematics methods course through the use of the LTLF materials were evident during a video case written analysis and group discussion that occurred at the end of the program with a different cohort. This is significant in that the PTs had not explicitly discussed the importance of these norms, nor were they reminded of the norms prior to the session. In addition, the PTs had participated in the prior video case discussions with at most three others, and thus had not constituted the norms as a group. Rather, the norms had been developed in four separate classrooms, yet appeared to re-emerge naturally during a similar discussion focused on analyzing teaching and learning.

To give the reader a further sense of what it means to exhibit the professional and sociomathematical norms examined in this study, we now turn our attention to a more detailed analysis of three norms that are particularly relevant to developing students’ mathematical understanding and to teachers’ continued professional development. We will first discuss results related to the professional norm of backing up claims with evidence—a norm that is critical to becoming a reflective practitioner. We then focus on the sociomathematical norms of naming,
labeling, distinguishing, and comparing mathematical ideas, and using mathematical arguments, both of which are critical to supporting students’ understanding of mathematics.

Providing Evidence

All seven of the PTs who participated substantially in the video discussion, as well as one who did not, made at least one claim during the discussion that they backed with evidence and/or reasoning, referring either to specific line numbers in the video transcript or directly referencing the transcript in another way (e.g., “on page 1 it says ...”). Of the 13 instances where PTs referenced the video transcript, 10 (77%) occurred without any prompting, three were prompted by the facilitator and one by another PT. Notably, all four PTs who were prompted to use evidence also had instances where they used evidence without prompting.

The following exchange illustrates how the practice of providing evidence to back claims was normative in the group:

Ken: Well, Arden’s [expression] works if you have his variable.
Jim: Exactly. So it works for all of them, if you use his variable, which he specified in the very beginning. But apparently the girls weren’t listening to what he said.
Ken: Where did he explain that? Because I—
Jim: Line 7. The equation was that $5n + 1$ equals the volume, and $n$ equals the length of one individual arm. So he told people, but everyone was just so caught up on the building number. [T326–332]

Here, one PT, Ken, actually prompts another to refer back to the transcript to back up his claim that a student had clearly defined his variable.

Yet another source of evidence of this norm is the use of line numbers in the PTs’ written work, which was completed before the group discussion, and thus before any prompting was provided. In this case, four of the PTs cited specific line numbers in their reflections, while another provided more indirect evidence. It is important to note that using evidence does not come naturally to PTs and, in fact, takes some time to develop in the first methods course.

Comparing Mathematical Ideas

All 11 of the PTs named, labeled, distinguished, and compared mathematical ideas either in the video discussion or their written reflections, providing strong evidence that doing so was normative in the group. Ten participated in the norm by directly comparing students’ thinking, and six, including the one who did not directly compare, made statements that indicated they thought comparing solutions was important.

In the video discussions, all eight of the participants who substantially contributed to the conversation exhibited this norm in at least one of three ways: (1) comparing their mathematical thinking to that of other PTs (“I was just going to say that I came up with the same formula, $5(n-1)$, but I saw it in a different way” [Lily, T65–66]); (2) comparing their thinking to the students’ in the videos (“They were adding 1 for that middle cube. They were kind of looking at it the same way I did.” [Evan, T338–340]); or (3) comparing the students’ thinking in the video to each other’s and/or recognizing that it was important that the teacher did so. The following, which immediately preceded the exchange with Abby and Jim cited above, illustrates this recognizing:

Well, a lot of it seemed to me like [the teacher]’s checking them for their own understanding. ... asking them to like compare and contrast is showing like if they understand their own method enough to talk about how it’s different from the way someone else did it, and how, how they’re the same. [Lew, T533–538]
In the written work, all but two of the PTs responded to at least one reflective prompt by discussing the different ways students in the video were defining their variable, which indicates that they were comparing students’ solutions as part of their individual analyses. Evidence of this norm was also shown in a variety of other ways in the written work, including recognizing that the teacher pushed students to compare their solution methods, directly discussing and comparing student solution methods, and discussing whether students really understood each other’s solutions. In addition, five PTs noted that comparing and contrasting solutions was important in their end-of-day reflection. For example, Roxy reflected that “It is important to tie everything together and see how or if different solutions are related to each other” [DS 8].

**Mathematical Arguments**

All of the PTs attempted to use mathematical explanations that consisted of a mathematical argument—not simply a procedural description or summary—in their own solutions to the problem [DS 1] and all but two either did so again or noted the importance of doing so when analyzing the student thinking or teacher moves in the video [DS 4–8, T]. In justifying their own solutions the PTs were successful in using a mathematical argument to varying degrees. To illustrate, consider Leah’s justification for her expression, $4(n - 1) + n$. “The solution supports the picture since you have four branches that are the size of the previous building $(n - 1)$, and one branch that is the size of the current building $(n)$” [DS 1]. Here, Leah justifies each part of her expression in relation to the diagram provided with the problem, though it would have been clearer if she had said “building number” instead of just “building.” In contrast, to justify her expression, $5n - 4$, Hana says, “My solution accommodates my visualization of 5 blocks adding every [time] to the original cube: one cube spreading out at its arms” [DS 1]. While Hana adequately justified the coefficient in her expression, she did not make any attempt to justify the $-4$, rendering her argument incomplete.

Further evidence that the behavior is normative can be seen in the fact that seven of the PTs made statements in their written reflections on the teacher and students in the video indicating that they recognized the importance of using mathematical argument. In the following reflection, for instance, Abby demonstrated the norm by noticing a lack of argumentation:

“One thing that stood out was that the teacher never asked Cassie to explain where the minus 4 came from. During her explanation, at the end she would just say, “and then you subtract 4.” There was no connection to the picture or explanation of where that came from. [DS 4]

In the end-of-day reflection that prompted for insights/connections related to teaching that the discussion generated for them, Iris wrote, “Teachers should ask questions that prompt connections between pictures and expressions/equations” [DS 8].

Similar statements related to this norm were made throughout the video discussion. In total, 24 instances of this norm were coded, involving nine different PTs. In one telling utterance, Roxy discusses how the students in the video were able to justify a part of a mathematical expression that the PTs could not in their own mathematical discussion:

“Yeah, because that’s what I had trouble seeing. I couldn’t figure out like how to describe where you take away the 4. ‘Cause I did it like Leah did it, with the 4—well, I did it in a table, but then I also saw the $4(n - 1) + n$. I was like, oh, well, that’s how you get your minus 4. But I like how [the solution looking at the minus 4 as subtracting the overlap when the middle is included in each “leg”] actually shows this is how you take away the 4. [T215–219]
Thus, Roxy recognizes the students’ mathematical argument, while at the same time indicating its importance by noting her own inability to fully justify a mathematical expression.

Conclusions

Professional and sociomathematical norms developed early in a teacher preparation program seem to be durable in the sense that they re-emerged in a similar situation at the end of the program. This is encouraging because these norms support a richness of discussion about the teaching and learning of mathematics that is not prevalent in descriptions of practicing teachers in research on teacher learning. In particular, the PTs in this study focused on student thinking and the implications of the teacher’s actions for supporting student thinking in a way that is not commonly seen. The fact that we were able to foster these professional and sociomathematical norms through use of a practice-based video case curriculum in a methods course (Stockero, 2008), combined with the findings from this study regarding the durability of the norms, suggests the value of making the development of such norms a key part of curricula used in university methods courses. Not only will moving PTs further along the teacher development trajectory during their university education give them a solid start, experiencing self-sustaining generative change will position them to accelerate their movement along the teacher professional development trajectory as they become more experienced teachers.

References


