MATHEMATICS TEACHER EDUCATORS' PURPOSES FOR K-8 CONTENT COURSES

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This report provides empirical findings from a study that examined the purposes of eight experienced mathematics teacher educators, who taught mathematics content courses for prospective K-8 teachers. The data revealed 15 common purposes, aligned to providing the opportunity to develop prospective teachers' pedagogical content knowledge and subject matter knowledge. Two of the purposes aligned with the pedagogical content knowledge (knowledge of curriculum and instructional strategies) are elaborated in this paper. Implications from this study contribute to the literature on mathematics teacher educators' purposes and provide insights into the teacher educators' classroom practices from the K-8 content courses.

INTRODUCTION

Research suggests many prospective teachers do not receive adequate experiences from their teacher education programs in order to develop deep, conceptual knowledge of the mathematics they will teach (e.g., Greenberg and Walsh 2008). As a result, the Conference Board of Mathematical Sciences (CBMS) (2012) proposed that all institutions preparing elementary teachers offer and require at least nine credits of mathematics content courses designed specifically for this population and focused on mathematical relevance, depth, and breadth, concluding that "teaching elementary mathematics requires both a wide range of pedagogical skills and considerable mathematical knowledge'' (p. 55). Research also suggests developing *pedagogical content knowledge* (PCK) and *subject matter knowledge* (SMK) is critical for prospective teacher education (An, Kulm, & Wu, 2004, Blömeke, Suhl, & Kaiser, 2014). In support of these efforts, various mathematics teacher education curriculum materials have been developed to help address these issues, however, very little research exists on what content (and how) are being taught in these courses, what PCK and SMK aspects are being emphasised and addressed, and what goals and purposes do the mathematics teacher educators (henceforth referred to as teacher educators) have in mind when teaching these courses.

Some research efforts have examined the development of mathematics teacher educators, their practice, self-studies, professional development, teacher educator collaborations, and more recently, the differences between the knowledge of K-12 teachers and teacher educators (e.g., Even, 2008; Goodell, 2006; Superfine & Li, 2014, Taylor, 2013; Tzur, 2001). Although these efforts represent a useful start, additional research and development work are needed in order to accumulate an empirical and conceptual knowledge base for mathematics teacher education. Superfine and Li (2014) recommend researching teacher educators' reflection on

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their practice and this could provide "insights into the potential mathematical and pedagogical purposes of those interactions" (p. 313). Building off of this recommendation, we present a conceptual depiction of teacher educator purposes for teaching K-8 content courses for prospective teachers based on empirical data collected during a case study of what eight teacher educators said during an initial interview. More specifically, we sought to understand, document, and investigate the research question: What purposes do experienced teacher educators have for prospective teachers to develop knowledge about teaching mathematics in a K-8 content course for teachers? We define *purposes* as what teacher educators want prospective teachers to learn from K-8 content courses. More specifically, the teacher educators' professional and personal intentions, that may or may not be included in the course syllabus and/or curriculum, for prospective teachers' specific knowledge development and learning outcomes.

THEORETICAL FRAMING FOR THE STUDY

Ball and Bass (2000) suggest that teachers' mathematical knowledge is important, and that "simply looking at the math problem or considering the content on which students are working does not lead to a sufficient appreciation of the specific mathematical knowledge or sensibility that it takes to teach that problem or that content" (p. 91). An, Kulm, and Wu (2004) also argue that prospective teachers' knowledge of pedagogy is especially important in mathematics teacher preparation programs. We frame this work in the research perspectives focused on teachers' knowledge development. In particular, this study is grounded in the perspective that aims at capturing teacher educators' purposes that support the development of prospective teachers' PCK and SMK. For the purpose of this paper, the focus of analysis is on teacher educators' purposes specifically supporting prospective teachers' development of PCK and SMK domains.

Pedagogical content knowledge, originally coined by Shulman (1986), is defined as teachers' knowledge about "the most useful ways of representing and formulating the subject that make it comprehensible to others" (p. 9). Grossman (1990) built on Shulman's work and identified four central domains of PCK: knowledge of curriculum, knowledge of instructional strategies, knowledge of students' understanding, and knowledge of assessment. Magnusson, Krajcik, and Borko (1999) further modified Grossman's perspective by adding a fifth element of PCK: orientation towards teaching.

In contrast, *subject matter knowledge* contains common content knowledge (i.e., math knowledge and skills used in professions other than teaching), knowledge at the mathematical horizon (i.e., awareness of mathematical connections between topics), and specialised content knowledge (Ball, Thames, & Phelps, 2008). The authors argue that specialised content knowledge is a critical domain of SMK, which entails the type of mathematical knowledge that is specifically unique to teaching and is "not

typically needed for purposes other than teaching" nor used in professions other than teaching (p. 400).

Accordingly, this work is framed in the perspective that teacher educators have various purposes aimed to provide the opportunities for prospective teachers to develop necessary knowledge bases, specifically attending to PCK, as well as SMK. It is critical that the teaching practices utilised by teacher educators, during teacher preparation courses, provide the opportunity for prospective teachers to develop the necessary knowledge that will enable prospective teachers to become effective mathematics teachers and successful educators.

METHODOLOGY

This study is a case study (Stake, 2005), where the "case" is a group of eight *experienced* teacher educator volunteers (5 males; 3 females) from five different universities in the Eastern portion of the U.S. who regularly teach content courses for prospective K-8 teachers. We define *experienced* as: a) having at least a Master's degree; b) having at least 20 years of K-12 teaching experience and teaching mathematics content to K-12 teachers; and c) being professionally active by attending/presenting at local, state, and national professional meetings. We treated the group of eight teacher educators as a single prototypical case, which allowed us to make claims about the nature of their purposes for teaching K-8 content courses as a whole. Data for the project were gathered through 1-hour semi-structured initial interviews, during which participants were asked about their educational background, their purposes for the K-8 content course they teach (i.e., intentions for small group and whole group instruction), whether (and how) explicit they were with prospective teachers about their purposes, and the approaches they used to engage prospective teachers to address the identified purposes. Interviews were audio-recorded, transcribed, and coded using constant comparison analysis (Corbin & Strauss, 2008).

A total of 326 codes emerged from the data analysis. They were arranged under 15 different purposes mirroring the knowledge domains of PCK and SMK. Instances of teacher educators' purposes were identified through interview responses as reflection on various tasks the teacher educators used to engage prospective teachers with course content. Two researchers independently coded each interview. Researchers met throughout the coding process to compare, verify, and finalise the codes. Through several iterations of sorting the purpose codes, a coding dictionary was created from the data to define and illustrate each purpose. The researchers collaborated to refine descriptions of specific purposes the teacher educators articulated.

RESULTS

Across the U.S., most content courses for prospective teachers are treated as a regular college mathematics course, hosted and taught by mathematics faculty in the mathematics department (Greenberg & Walsh, 2008). As we analysed the data from the classrooms of experienced teacher educators, we noticed numerous K-8

connections with regard to students' learning, curriculum, and classroom connections. In fact, we identified 15 different purposes that experienced teacher educators utilised via these K-8 connections in effort to develop prospective teachers' PCK and SMK. For the purpose of this paper, we present the results from two (out of 15) purposes related to PCK: 1) knowledge of instructional strategies; and 2) knowledge of curriculum.

Know about instructional tools used in K-8 teaching

Teacher educators mentioned they desire for prospective teachers to know about instructional tools used in K-8 teaching—a purpose that addresses the *PCK component of providing the opportunity for prospective teachers to develop their knowledge of instructional strategies*. That is, they incorporate models, physical manipulatives, and representations in their content courses to articulate mathematical concepts studied and taught at the K-8 level. A common theme among the teacher educators was that in their content course, they wanted to provide the opportunity for prospective teachers to develop multiple approaches, representations, and tools for learning and teaching mathematics. They indicated they used instructional tools (i.e., physical models and/or manipulatives) to help prospective teachers make better conceptual connections of mathematical concepts and become familiar and comfortable with using the tools in their future classroom. All study participants perceived the use of manipulatives as an integral part of the mathematics content courses. For example, one teacher educator shared, "I want the [prospective teachers] to come away [from the course] understanding the power of physical models…that they feel comfortable in seeing how to use those physical models when they're working with kids" [Ian]. Similarly, another teacher educator commented,

> I think [prospective teachers] get the message that the answer isn't always good enough. They realise they're going to be teaching children. They're going to have to be explaining things. They're going to need a deeper understanding. They kind of get that, and so they seem to get the message by the end [of the course] that the process of being able to explain "what and why" is what's important… and that manipulatives can provide a visual for helping them explain the "what and why" to their students. [Trina]

Every teacher educator in our study mentioned his/her personal and professional intentions and purposes for their students to be well equipped mathematically and pedagogically for K-8 teaching, in which (the teacher educators believed) that K-8 models and manipulatives play a critical role. Furthermore, they indicated that they primarily used K-8 instructional tools to extend prospective teachers' mathematical thinking to go beyond "the answer," to model and make better sense of mathematical concepts, and to be able to construct more accurate and thorough mathematical explanations and justifications of their work.

Expose to policy documents on curriculum, content, and teaching

Teacher educators indicated that they want to expose prospective teachers to K-8 standards and policy documents on curriculum, content, and teaching—this purpose

aligns to the *PCK component of providing the opportunity to develop prospective teachers' knowledge of K-8 school curriculum.* Two different types of K-8 curriculum connections were articulated by the teacher educators. One, teacher educators discussed the scope and sequence (i.e., specific grade bands) of where prospective teachers might encounter the mathematical topics they were learning during the content course in K-8 school curriculum. For example, one teacher educator talked about division of fractions and the grade levels at which this topic is typically introduced to children. He verbalised,

> We mainly talk about how the content we're covering relates to what the students they are going to have in class have to do. For example, we talk about how modelling the division of fractions is actually something that appears in 5th grade, so [school children] are going to be asked to do these things that we are doing in class. [Oliver]

The second K-8 connection the teacher educators articulated was that they specifically addressed mathematics teaching practices described in K-8 standards documents. They wanted prospective teachers to know key processes and proficiencies for the type of mathematical thinking and reasoning K-8 students should engage in, which teacher educators modelled for them during content courses. The teacher educators mentioned several documents that helped them to make these connections: *Standards for mathematics practices* (CCSS, 2010); *Principles and standards for school mathematics* (NCTM, 1989); and *Adding it up: Helping children learn mathematics* (NRC, 2001). Teacher educators shared that they either directly referenced these documents or selected a few focal points from the documents to discuss with prospective teachers. For example, one teacher educator stated,

> I [want] my students to be familiar with the NCTM Process Standards and now the Standards for Mathematical Practice of the Common Core…I love the new buzzword of "sense making." I am somewhat explicit with them about that. [I say to them that] math makes sense, math had better make sense, and it had better make sense to you if you're going to teach it to kids. [Ethan]

Teacher educators also indicated that these documents play a dual purpose in their content courses: a) they help to unveil and put forth a few "practical" suggestions to the prospective teachers about teaching and learning K-8 mathematics, and b) they help teacher educators to model the methods and practices (described in these documents) directly with prospective teachers. Teacher educators shared that they structure the learning opportunities in their content courses to specifically address these standards through course activities so that their prospective teachers are able to experience the mathematical learning echoed in these documents firsthand.

In the study, a total of 15 different purposes (i.e., teacher educators' personal and professional intentions) were identified from interviews, which indicated classroom opportunities for prospective teachers to develop PCK or SMK during mathematics content courses (see Table 1).

Teacher Educator's Purposes for Developing PCK for Prospective Teachers is to:	Teacher Educator's Purposes for Developing SMK for Prospective Teachers is to:
Know about instructional tools used in K-8 teaching	Understand mathematical concepts at a deeper level and articulate the why behind the concepts and formulas
Expose them to policy documents on curriculum, content, and teaching	Develop multiple ways and/or approaches to solve mathematical tasks
Know about K-8 experiences/experiences K-8 students have	Have concrete experiences (e.g., manipulatives) to develop conceptual understanding of mathematical concepts
Experience mathematical success and confidence	See mathematics conceptually
Change their attitude to a positive one towards the subject of mathematics	Experience mathematical learning in different ways
Change their attitude towards teaching math	Know K-8 mathematical concepts they will teach
Have fun with math and see that math can be fun	Develop and improve their mathematical explanations and language
Engage in collaboration	

Table 1: Summary of teacher educators' mathematical and pedagogical purposes for teaching K-8 content courses

CONCLUDING REMARKS

Results indicate that teacher educators not only focus their content courses on developing the mathematical knowledge of prospective teachers, but on providing the opportunity for prospective teachers to develop four components of PCK: knowledge of curriculum, knowledge of instructional strategies, knowledge of students' understanding, and orientation towards teaching. The experienced teacher educators used the processes of reconceptualising, revisiting, revising, and re-learning mathematics in the course while making connections to K-8 students' learning, teaching, and curriculum as channels to develop prospective teachers' PCK. We did not find any data indicating that teacher educators provided the opportunity to develop prospective teachers' knowledge of assessment—a fifth component of PCK.

The highlighted purposes are representative of eight experienced teacher educators' reasons for engaging prospective teachers in various mathematical learning experiences throughout K-8 content courses. These 15 purposes may help teacher educators of all experience levels to design, plan, and teach courses for prospective teachers. The list of purposes may not be exhaustive and may vary across different

settings based on the experiences of teacher educators; however, these empirical data provide a foundation on which other teacher educators may build their practice.

With this study, we join others (e.g., Superfine & Li, 2014; Taylor, 2013) in providing new insights into the knowledge and purposes that teacher educators draw on to enrich the learning experiences of prospective teachers. Ultimately, this study serves as a window for engaging teacher educators in professional conversations about specific purposes embedded in the teaching of content courses to explore further questions: (a) What PCK connections are critical to make in the content courses? (b) What SMK connections are essential to address in content courses for prospective teachers? and (c) How do we help university faculty (especially the non-educators) in making these PCK and SMK connections in the content courses?

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