FACILITATING PROSPECTIVE TEACHERS' KNOWLEDGE OF STUDENT UNDERSTANDING: THE CASE OF ONE MATHEMATICS TEACHER EDUCATOR

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As a field, we know little about the practices of mathematics teacher educators, especially in relation to developing pedagogical content knowledge (PCK), as these practices are not widely researched or disseminated. This study investigated the actions and related purposes of what one mathematics teacher educator said and wrote during whole group instruction across three years in her elementary mathematics content/methods course to provide opportunities for prospective teachers to develop knowledge of student understanding. Findings from this study contribute to the literature on practices of teacher educators that can inform the design and implementation of teacher preparation programs.

INTRODUCTION AND BACKGROUND

During the past 25 years, there has been increased attention to conceptualizing the knowledge bases that mathematics teachers need to develop in order to effectively teach mathematics. Shulman (1986) identified a variety of knowledge bases and introduced the idea of pedagogical content knowledge (PCK). Subsequently, other researchers have built on this work in a variety of disciplines, including English (e.g., Grossman, 1990), science (e.g., Magnusson, Krajcik, & Borko, 1999), and mathematics (e.g., Ball, Thames, & Phelps, 2008).

An, Kulm, and Wu (2004) have argued that enhancing prospective teachers' PCK "should be the most important element in the domain of mathematics teachers' knowledge" (p. 146). Thus, it follows that a significant focus of mathematics teacher educators should be to provide opportunities for prospective teachers to develop PCK. There has been great attention to conceptualizing and delineating the components of PCK, yet there is little research that investigates how mathematics teacher educators facilitate its development in prospective teachers. In fact, we know little about the practices of mathematics teacher educators as these practices are not widely documented or disseminated (e.g., Bergsten & Grevholm, 2008).

There have been some research efforts related to the work of mathematics teacher educators (henceforth referred to as teacher educators). For example, teacher educators have conducted self-studies of their practice (e.g., Tzur, 2001), investigated professional development opportunities for teacher educators (e.g., Even, 2008), described specific activities that they use in their methods courses (Goodell, 2006), and examined a collaboration between novice teacher educators and a more experienced teacher educator (Van Zoest, Moore, & Stockero, 2006). Although these

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efforts represent a good start, additional research and development work are needed in order to accumulate a useful knowledge base for mathematics teacher education.

To better understand the teaching of university mathematics content/methods courses, this research study aimed to investigate the research questions: (a) What actions does an experienced and U.S. nationally recognized mathematics teacher educator enact during whole group instruction in an elementary mathematics content/methods course?; and (b) For what purposes does she use the identified actions? *Actions* are defined as what the teacher educator said and wrote (i.e., on the board, overhead, or document camera) while instructing prospective teachers. In this paper, empirical data is presented in relation to actions one teacher educator employed to provide the opportunity for prospective teachers to develop knowledge of one PCK component—*knowledge of student understanding*. Whether prospective teachers' developed *knowledge of student understanding* was not the focus of the study—rather, the focus was on understanding one teacher educator's practices as she provided the opportunity for prospective teachers to develop this knowledge.

THEORETICAL FRAMING FOR THE STUDY

Within the context of mathematics content/methods courses, teacher educators employ various actions aimed to improve prospective teachers' knowledge bases (e.g., content knowledge, PCK, general pedagogical knowledge, etc.). One significant focus in such courses is often providing the opportunity for prospective teachers to develop what Shulman (1986) called PCK. Building on Shulman's work, Grossman (1990) identified *knowledge of students' understanding, conceptions, and misconceptions of particular topics in a subject* as one of four central components of PCK. Magnusson, Krajcik, and Borko (1999) further modified this component by including *teachers' knowledge of student misconceptions, approaches, and strategies when subject specific concepts are addressed*. Ultimately, this PCK component—which is the focus of this study—includes knowledge prospective teachers must have about students in order to help children develop specific mathematical knowledge. This includes strategies that will help prospective teachers identify students' conceptions and misconceptions for solving various types of mathematical problems, as well as strategies that will help them aid students in understanding mathematical concepts and learning about specific mathematical topics.

METHOD

A single case study design was used because the research questions were exploratory in nature and enabled the author to try "to illuminate a decision or set of decisions" (Yin, 2003, p. 12) regarding actions specific to teaching prospective teachers that one teacher educator used in her mathematics content/methods course. This design was used not because of an interest in the specific case participant, but in understanding actions specific to teaching prospective teachers that teacher educators use to provide the opportunity for prospective teachers to develop PCK. In other words, the case is examined to provide insight into a larger issue (Stake, 2005).

This study was conducted over a three-year period in the same course at a Midwestern U.S. university that was the second of a two-course sequence required for certification in elementary education (grades 1-6). The focus was on the content and complexities of teaching geometry, measurement, probability, and statistics. The course met for 75 minutes twice a week for 15 weeks, and prospective teachers enrolled in the course during their final semester of coursework prior to a year-long student teaching placement. The average class size for the three years was 24 students (22 females and 2 males).

Participant

Leah (a pseudonym), a former elementary school teacher and the teacher educator chosen for the case, taught the methods course six times over her seven years as a teacher of teachers by the conclusion of the study. She was purposeful about what she said and did in her classroom—devoting a tremendous amount of time each week to reflect on how her instruction influenced her students' participation and performance. Leah had received national recognition for her teaching, service, and scholarship since receiving her doctoral degree. In addition, her students recognized her efforts by consistently rating her teaching performance at the highest levels in course evaluations. Leah was willing to open her classroom and her teaching for others to learn from as evidenced by her mentoring of doctoral students who taught the same course. She frequently invited these students into her classroom to observe her teach and engage in weekly planning sessions, and to ask her questions during debriefing sessions related to what she said and did during her class. Her mentoring experience, as well as her experience as an established teacher educator, made her particularly suitable to explore the research questions.

Data collection and analysis

Data included videotaped mathematics content/methods course lessons taught by Leah in Spring 2007 and field notes taken by the author in Spring 2008 and Spring 2010. This data was initially collected for another purpose. Snapshots of Leah's practice (i.e., what she said and wrote) were identified in the data. In the videotapes, a snapshot ranged in length from 10 seconds to 3 minutes and entailed: (a) an entire speaking turn from Leah, (b) a segment of a lengthy speaking turn from Leah, or (c) dialogue between the prospective teachers and Leah around a single mathematical concept. Snapshots identified in the field notes included an image of: (a) what Leah wrote or drew on the board, or (b) what Leah said related to providing an opportunity for prospective teachers to develop PCK. Additional sources of data included interviews where Leah provided commentary on three extended video segments from Spring 2007 about her purpose(s) and/or what she was hoping to address in class about learning to teach, as well as video/field note based interviews where Leah discussed her purpose(s) for employing actions identified in preselected snapshots.

The author conducted all analyses of the data. The HyperResearch qualitative data analysis software program (ResearchWare, 2007) was used to code Leah's PCK-related actions captured in the video and field note snapshots. Snapshots tagged during the initial coding were categorized (and re-categorized) using Magnusson et al. (1999) conceptualization of *knowledge of students' understanding* as including teachers' knowledge of student misconceptions, approaches, and strategies when subject specific concepts are addressed. Through the categorization process, themes of actions within *knowledge of student understanding* began to emerge and descriptions of those themes were written. Through several iterations of sorting the snapshots, a coding dictionary was created from the data to define and illustrate each action. The researcher collaborated with Leah to refine descriptions of specific actions, as well as the categories of actions she employed to provide the opportunity for prospective teachers to develop *knowledge of student understanding*. Consistency of coding was verified with three other researchers.

RESULTS

In the study, seven actions were identified, grouped into four major categories, aligning to what Leah said or did to help prospective teachers develop *knowledge of student understanding*. Table 1 summarizes the actions. Below, one of the most prevalent actions (bolded in Table 1) along with corresponding purposes, is elaborated on to provide the reader with specific images of Leah's practice.

Categories of student understanding	*Actions*
Sample grade 1-6 student mathematical answers	States atypical grade 1-6 student answers/thinking to mathematical concepts under discussion
	States incorrect grade 1-6 student answer to mathematical concept under discussion that relays the student's incorrect mathematical understanding about the topic under discussion
Predicting grade 1-6 student mathematical responses	**Prompts prospective teachers to predict mathematical answers/strategies grade 1-6 students generate/how grade 1-6 students will solve posed mathematical problems (which were posed to the prospective teachers or prospective teachers discussed as whole class)**
Grade 1-6 student mathematical misconceptions or error patterns	Shares misconceptions and/or error patterns grade 1-6 students (and teachers) have regarding mathematical concept(s) under discussion
Mathematical concepts that are abstract or confusing for grade 1-6 students	Articulates mathematical concepts that are abstract for grade 1-6 students
	Articulates language issue grade 1-6 students may have that interfere with grade 1-6 students' understanding the mathematics under discussion
	Describes examples of mathematical connections that grade 1-6 students may not make

Table 1: Summary of teacher educator actions identified to facilitate the development of prospective teachers' knowledge of student understanding

Throughout the semester, one of Leah's most prevalent actions was to ask prospective teachers to *predict what grade 1-6 students will say* to a mathematical problem or concept currently under discussion in the class. This action was enacted in a variety of ways. Sometimes, Leah had her students engage in the mathematical task themselves and then predict typical grade-level responses for the mathematical situation. For example, Leah gave the prospective teachers three squares and asked them to create all of the different possible shapes with the three squares where the side of one square must be flush with the side of another square (i.e., the corner of one square may not solely touch the corner of another square nor may just part of a side of one square solely touch a part of the side of another square). Then the prospective teachers worked in groups to find the number of different shapes they could make with four squares (given specified constraints). Leah then prompted them to make a prediction for how many different shapes were possible using five squares. She said,

> Ok so now we [haven't done] this problem, but I want a prediction. So kids often are looking for patterns, so we are going to have some guesses for how many shapes we are going to find with five squares. What are your guesses and predictions? So you are not actually finding them yet, you are just guessing. [Video Day13]

At this point, there was a whole class discussion where prospective teachers made predictions about how many different shapes could be formed using five congruent squares. The prospective teachers started by stating their predictions, but then Leah prompted them to provide answers they thought elementary students might pose. The predictions, with justification, they provided included: (a) 10 (doubled five); (b) six (difference of one); (c) eight (difference of three between the two and five in the "number of different shapes" column, so add the difference of three to the five); and (d) seven (2 + 5 in the "number of different shapes" column). However, the prospective teachers did not provide all of the answers and describe all of the patterns that Leah had observed children provide. The prospective teachers were still missing two responses that grade 5 students gave when she had posed this problem to them. Leah decided that she would state these two additional patterns. She commented,

> You are missing a couple other ones that kids come up with…There are two right answers in my brain that you are trying to figure out…I will just tell you. Ok, $3 + 2 = 5$, so $4 + 5 = 9$. Ok? This is a sophisticated child; let me see if I can remember it—$2 \times 2 + 1 = 5$ so $5 \times 2 + 1 = 11$. Yes, sign that kid up. That kid is doing some thinking. So, this one wasn't in the chart and says if I double two and add one I get five, so I am going to double five which is 10 and add one which is 11.

Leah also asked prospective teachers to predict grade 1-6 student answers in other contexts. For example, she asked prospective teachers to predict what a student would say when she engaged them in analyzing student work—specifically, "What do we hope here" when the subtraction problems 25 – 21 and 103 – 99 were posed [FN2008 Day13]. Here, Leah shared that she would expect a child to solve these two subtraction problems without using the traditional algorithm. She also shared the results of a research study where children were given two rectangles and asked to find the perimeter of each figure. One rectangle had two adjacent sides labelled while the other rectangle had all four sides labelled. Leah asked the prospective teachers to predict what elementary students would do [FN2010 Day14].

Occasionally, when Leah asked her students to *predict what grade 1-6 students will say*, after they worked on mathematical tasks, analyzed student work, etc., she reiterated the potential student answer suggested by a prospective teacher. An example of this was seen when she engaged prospective teachers in the game Roller Derby—a game where two dice are rolled, the sum is computed, and if an individual had a counter on the number of the sum that was rolled, one counter was removed. She asked the prospective teachers to predict where elementary students would put their 12 counters on the game board that had the numbers 1 through 12. She stated,

Leah: So let me ask you this question, I have now done this game in Kindergarten, first, second and third grade. What do you think the number one answer is? …How kids put their distribution down?

Student: One on each.

Leah: They put one on each. [Video Day2]

Other times, Leah carried out the action by answering her own question before the prospective teachers had an opportunity to do so, as seen in the next example.

Leah: Some of the big measurement ideas that you need to have, these are kind of your goals and things you are assessing. First you must include a number and a unit…You may compare two measurements if the same unit is used. So, if I say that this table is this line here and that is 30 inches and this table here is five feet. Which one is longer? The five feet or 30 inches?

Student: Five feet.

Leah: But what are the kids going to say? They are going to say this one because the number is bigger. Now, I cannot make those comparisons because my unit is different. These are the big ideas of measurement that take a lot of time to develop. [Video Day19]

Leah purposefully *prompted prospective teachers to predict what grade 1-6 students will say* because she wanted them to keep thinking about children when they plan and teach lessons. Leah admitted that many times when she asked her students to predict what grade 1-6 students would say, the question was rhetorical, but she kept posing the questions because they need to keep thinking about children. Leah elaborated on her observations of lessons that prospective teachers teach in their university field experience in local elementary schools while they are enrolled in her course:

> They're so focused on themselves that they don't think about kids. And they focus on what they're going to do. And I try and get them to think about the kids. And so don't just think about the questions you pose, think about the answers you're going to get and how are you going to respond to those answers. Now with no teaching experience, I

> recognize that they're not very good at this. They don't know what to expect…but I keep throwing [those questions] out there. [EVT#3]

Leah recognized that prospective teachers lack experience in elementary classrooms, but she reiterated that she wanted them to consider students in addition to their own teaching practice, emotions, or beliefs. A second purpose Leah communicated for sharing predictions grade 1-6 students articulated was that she "want[ed her students] to know that kids are very creative and come up with lots of interesting things that [the prospective teachers] won't anticipate" [VDFNInt1].

As the examples above illustrate, Leah provided opportunities for prospective teachers to predict grade 1-6 student responses in various ways. Her overarching purpose for employing each of the observed strategies was to encourage her students to think about children and what children will say and do (i.e., prospective teachers' knowledge of approaches and strategies grade school students use) so that they would be more apt to be responsive to students during their own instruction.

CONCLUSIONS

The results discussed above detail one of the seven identified actions, and two corresponding purposes, that Leah demonstrated to provide the opportunity for prospective teachers to develop *knowledge of student understanding*. Five different themes were identified across three years regarding how Leah carried out this one action in her classroom, highlighting the complexity of the work of supporting mathematics teacher learning. Although the identified actions and purposes may not be exhaustive and may vary across different settings (e.g., at other grade levels such as middle and secondary), based on the experiences of teacher educators (e.g., teaching experience at the level in which they are preparing teachers to teach, etc.), they provide a foundation on which other studies can build. Thus, this study joins others (e.g., Even, 2008; Van Zoest, et al., 2006) in providing new insights regarding practices that might enrich the preparation of mathematics teachers—practices teacher educators could draw upon in order to enrich the learning experience of prospective teachers.

Specifically, findings from this study build on research focused on identifying components of PCK that prospective teachers need to develop to identify actions teacher educators can use to develop that PCK. The seven actions listed above may help teacher educators plan instructional activities to engage prospective teachers in opportunities to foster their *knowledge of student understanding*. Additionally, the examples of how one of the actions was implemented may help teacher educators consider additional ways to challenge and expand prospective teachers' initial understandings of what it means to effectively teach mathematics to grade 1-6 students. The highlighted purposes are representative of Leah's thinking about one action, which begins to develop a sense of why one might demonstrate specific actions in teaching prospective teachers. Finally, this research serves as a model for extending work on PCK from teaching at the K-12 student level to the teacher education level, conceptualizing actions and purposes teacher educators need to consider to develop PCK.

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