Handbook of Research on Teacher Education and Professional Development

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Chapter 31
Defining Effective Learning Tasks for All

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ABSTRACT

An effective mathematics program may be defined as one in which classroom teachers implement tasks and activities that allow all students opportunities to engage in high levels of mathematical thinking and reasoning (NCTM, 2014). In the chapter, we describe background information regarding the preparation of practicing and prospective teachers when implementing research-based practices in the inclusive classroom. Specifically, we provide explicit background information from the extant literature regarding: 1. Equity, 2. Universal Design for Learning, and 3. How to use games as classroom activities to promote the development of mathematical concepts, skills, and conceptual reasoning.

INTRODUCTION

An effective mathematics program may be defined as one in which classroom teachers implement tasks and activities that allow all students opportunities to engage in high levels of mathematical thinking and reasoning (NCTM, 2014). However, many teachers claim they are ill prepared when asked to meet the needs of students in inclusive classrooms (Rose & Meyer, 2000; Spencer, 2011). A key factor that negatively impacts students’ mathematical development is the contrast that often exists between the needs of individual students and the type of instruction received (Buchheister, Jackson, & Taylor, 2014; Kroesbergen & Van Luit, 2003). While each and every child that enters a classroom should be provided the “opportunity to reach his or her potential, the current education system does not adequately address these needs. [In fact], the traditional methods used by teachers often focus on exposing and remedying

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deficits; setting up some students for a pattern of failure” (Subban, 2006, p. 938). Classrooms today reflect widespread diversity including students with disabilities, students exceeding grade level expectations, students from various cultural backgrounds, and students whose home language is not English (Subban, 2006), and unfortunately, many general education classroom teachers have a dearth of knowledge regarding specialized practices that provide students access to high quality mathematics instruction (Macinni & Gagnon, 2006). Consequently, it is integral that professional development and teacher education programs help teachers acquire knowledge and skills necessary to provide all students—regardless of mathematical understanding, language proficiency, or cultural experience—with the greatest opportunity to learn.

Universal Design for Learning (UDL) defines a framework to aid practicing and prospective teachers in providing access and opportunity to high quality mathematics. In addition, with many variations and various mediums, mathematical games naturally provide multiple modes of presentation and expression while simultaneously engaging and motivating students to participate in discussions of key mathematical ideas. Thus, games, as viewed through a UDL lens, are tasks that provide learning for all. In this chapter, the authors describe how the theory of UDL may be incorporated into the general education classroom through the use of mathematical games as lesson activities. Game play not only provides a context for students to use critical thinking skills that reflect mathematical proficiencies (e.g., communication, modeling, quantitative reasoning), but the strategic implementation of games allows for multiple entry points so students with a wide range of mathematical experience are empowered to participate in the problem solving process (Jackson, Taylor, & Buchheister, 2013). With knowledge of effective planning strategies through the integration of UDL (Courey, Tappe, Skiker, & LePage, 2012), teaching and learning of mathematics would embody the elements of equitable instruction. The authors conclude the chapter with practical implications for teacher education, professional development, and subsequent research to further support the teaching and learning of equitable mathematics using UDL as a framework to define learning tasks through game play.

**Equity in Mathematics Education: What Is It?**

Equity has ranged in meaning from issues of access to making content culturally relevant to disrupting structural norms (DiME, 2007; Gates & Jorgensen, 2009). Equity may also be viewed as a process or as a product (Crenshaw, 1988; Gutiérrez, 2002; Martin, 2003; Rousseau & Tate, 2003). Essentially, seeing equity as a process means treating all students equally, without regard to race, ethnicity, or economic background. On the other hand, seeing equity as a product means differentiating instruction based upon students’ needs in order to promote equal learning outcomes. Differentiated instruction is a teaching strategy designed to recognize and address students’ learning preferences, strengths, and weaknesses (Subban, 2006), which should be a standard “component for teachers’ professional development in order to maximize effectiveness” (Chen & Herron, 2014, p. 24). Thus, implementing this level of instruction requires the dedication and knowledgebase of a well prepared teacher (van Garderen, Scheuermann, Jackson, & Hampton, 2009; Morgan, 2014).

The authors adopt the view of equity as a product, and define teaching mathematics for equitable outcomes as approaches to teaching mathematics that are respectful of students’ ethnic, racial, and economic backgrounds that promotes equal learning outcomes. More specifically, the authors draw on Gutiérrez’s (2007; 2009; 2012) definition of equity, which include access (i.e., resources that provide students an opportunity to learn and participate in the learning of mathematics), achievement (i.e., student outcomes), identity (i.e., drawing on students’ cultural frame to see themselves and the broader society
in mathematics), and power (i.e., social transformations such as voice, who is being privileged, and using mathematics to read and change the world). Gutiérrez argues equity encompasses the power of not predicting students’ outcomes (e.g., participation, achievement, ability) based on their background, race, class, or gender. Instead, equity embodies how students are positioned and the power they are given in the mathematics classroom as their identities evolve through discourse and social interaction.

The National Council of Teachers of Mathematics (NCTM) asserts that in order to teach in an equitable manner, teachers and schools must maintain “high expectations and strong support for all students” (NCTM, 2000, p. 11), meaning that mathematics teachers must provide opportunities for students to learn challenging mathematics regardless of their students’ “personal characteristics, backgrounds, or physical challenges” (p. 12). Moreover, to address issues of equity, mathematics teachers must include aspects of the student’s culture and language to support mathematics learning for all students (NCTM, 2014).

Some mathematics teachers have been unsuccessful in supporting students from cultural and linguistic backgrounds to achieve in mathematics (Suleiman, 1997), and maintaining the status quo will only continue to “widen the gap between teachers and children in schools” (Sleeter, 2001, p. 96). Yet, many mathematics teachers contend issues of equity are not relevant factors in the mathematics classroom because they view mathematics as a universal, culture-free subject (Rousseau & Tate, 2003). However, there is a growing body of mathematics education researchers who understand mathematics and mathematical knowledge are neither universal nor culturally neutral, but are situated in a sociocultural framework (Ukpokodu, 2011). Gay (2000) argues that if we “decontextualize[] teaching and learning from the ethnicities and cultures of students [it] minimizes the chances that their achievement potential will ever be fully realized” (p. 23). Thus, transitioning to an equity-centered paradigm in mathematics education requires that the mathematics education community “value the cultural and lived experiences of all children…[and] the belief that all children possess strong intellectual capacity and bring a wealth of informal, out-of-school knowledge to the teaching and learning process” (Lemons-Smith, 2008, p. 913). It is imperative that mathematics teachers are given the opportunity to develop this equity-centered orientation toward mathematics teaching and learning to effectively instruct all students.

In order to effectively accomplish the goal of teaching mathematics from an equity stance, teachers must understand that students from diverse backgrounds come into the mathematics classroom with different world-views. Additionally, teachers must be willing to move beyond teaching mathematics from a Eurocentric viewpoint and practice pedagogy related to equity by building relationships, setting high expectations, and helping students maintain their identities (Ladson-Billings, 1994; 2001; Malloy 2002; 2009; Matthews, 2005).

**Universal Design for Learning**

Universal Design for Learning is an equitable approach, which serves as a planning framework teachers may use to analyze the structure and content of the curriculum in order to embed appropriate accommodations (Subban, 2006) that meet the needs of students with a broad range of interests and skills. The principles of UDL are grounded in research of learner differences and highly effective pedagogy (Israel, Ribuffo, & Smith, 2014; Jimenez, Graf, & Rose, 2007); and thus support the premise of equitable instruction and provide a framework that educators may use to develop instructional activities that are appropriate and challenging for all students. While the initial focus of the theory attended to students with special needs, it is important to recognize that analyzing activities and planning through the UDL framework benefits all students.
It is compulsory that effective teacher preparation programs and professional development reflect empirical evidence that responds to the needs of an ever-changing population of students and dedicates explicit methods to provide prospective and practicing teachers with the knowledge and skills necessary to successfully instruct diverse students. By anticipating potential barriers, classroom teachers may embed instructional strategies and scaffolds that support students’ needs; thus, providing opportunities that allow all students, including those who struggle with mathematics, access to the general education curriculum (Buchheister, Jackson, & Taylor, 2014; Jimenez, Graf, & Rose, 2007). The three components of UDL include multiple modes of: (a) presentation (e.g., multiple modes of representation), (b) expression (e.g., sharing thinking through various student-selected modalities), and (c) engagement (e.g., incorporating students’ strengths and interests) (Basham & Marino, 2013; Israel, Ribuffo, & Smith, 2014). Each tenet of UDL focuses on the how (expression), what (representation), and why (engagement) of learning, which are the central tenets of the network of the brain (National Center on Universal Design for Learning, 2014).

Incorporating multiple means of presentation into the lesson supports students by representing mathematical content in various modes including discussions; stories, songs, or poems; virtual manipulatives; concrete objects; or real world contexts. Representations “embody critical features of mathematical constructs and actions” (NCTM, 2014, p. 24) and serve as tools to externalize, share, and preserve one’s understanding of mathematical ideas (NRC, 2001). Moreover, working through multiple representations of mathematical ideas promotes sense-making as students analyze, evaluate, select, and use multiple representations (NCTM, 2014). However, traditional mathematics texts or activities often represent content through equations or word problems that seem irrelevant to the lived experiences of children in the classroom (Polikoff, 2015). As a result, students often perform isolated drills and practice without ascribing deep meaning to the operations or analyzing the mathematical context underlying the word problem. The initial barrier is within the representation of the mathematics and the expectation of a specific interpretation of the mathematical convention; thus, instruction may be negatively affected. However, when content is presented through a variety of instructional materials and represented through various mediums, such as games, students have the opportunity to not only conceptualize the content in a way that corresponds to their learning preference and previous experiences, but this method of integrating multiple representations of mathematical content through modalities including hands-on experiences, illustrations, diagrams, or audio and visual supports also provides students with opportunities to translate the content across multiple representational forms; thus developing a deeper conceptual understanding of the underlying mathematics (Taylor, Buchheister, & Jackson, under review). For example, when students share strategies for determining the number of “bears in a cave,” a missing addend problem, students may use concrete objects, pictorial representations, algebraic equations (both addition and subtraction), and oral language to solve the contextual situation.

Universal Design for Learning not only includes various ways to present content to learners, but it also encourages the learner to express his or her understanding through means other than traditional pencil and paper formats such as mathematical worksheets or practice pages filled with computational exercises. Solely providing students with opportunities to express mathematical knowledge through a single mode may be a barrier to effectively evaluate students’ progress and understanding. An integral component of learning involves the student’s construction of representations to express ideas. As van Scy (1995) argues, these expressions define learning opportunities because the learner may choose from a variety of modalities (e.g., words, pictures, numbers, language, technology) to clarify their thinking. Moreover, as students record their thinking and reasoning it allows them to re-examine their ideas and revise their
understanding. When they are attempting to meet a particular goal in the context of game play, they find ways to describe their strategies, communicate observed patterns, and compare relationships. Allowing students to present their mathematical thinking through various mediums (e.g., words, numbers, pictures, tangible objects) supports effective evaluation and developmentally appropriate practice by promoting individual appropriateness through respect for individual students and their unique learning needs.

Finally, supporting the mathematical development of all learners also includes attending to the students’ affect. By considering how to effectively engage a diverse population of students through multiple means of engagement, teachers provide methods that promote interactions that empower students in the learning process. In the following section, the authors describe how the three elements of the UDL framework may be used as a foundation to analyze mathematical games and subsequently ensure that students in early childhood and elementary grades have access to the general education curriculum by developing robust learning experiences (Jimenez, Graf, & Rose, 2007) grounded in rigorous mathematical content.

**GAMES IN THE MATHEMATICS CLASSROOM**

van Oers (1996) argued that the socio-cultural context of mathematical instruction was vital to mathematical learning, and that students should be motivated to participate, not only because the activity is of interest to them, but also because the task is open so that students with a range of mathematical experiences may be engaged. Mathematical games embody each of these assertions. Even with rules, games are flexible and embody multiple variations to accommodate students’ individual needs’ and interests, especially when integrated with discussion questions that encourage reflection and representation (Buchheister, Jackson, & Taylor, 2015; Dockett & Perry, 2010; Jackson, Taylor, & Buchheister, 2013). Consequently, as students engage in game play, they are building and extending their mathematical language and communication through rich mathematical discussions that occur during the game, and they are engaged in critical thinking related to specific mathematical concepts (Jackson, Taylor, & Buchheister, 2013). Games may be used as a forum through which classroom teachers may “look at or listen carefully to the talk, the writing, and the actions through which pupils develop and display the state of their understanding” (Black & Wiliam, 1998, 143). And, knowing how students view mathematical ideas provides a solid foundation on which classroom teachers may develop questions and tasks that will expand the mathematical understanding of all students.

Moreover, games provide opportunities for all students to engage in the learning of mathematics because games have multiple entry points and incorporate the use of multiple strategies so all learners may participate. Smith and Backman (1975) argued that games may develop mathematical concepts, improve perceptual abilities, and encourage problem solving and logical thinking. When teachers use mathematical games that are:

1. Grounded in mathematics,
2. Self-directed and engaging, and
3. Appropriate and challenging to all students, they provide students with opportunities to extend their mathematical reasoning and understanding (Jackson, Taylor, & Buchheister, 2013).

More specifically, games provide multiple means of presentation to support the ways in which meaning is assigned to what we see and recognize (i.e., what we learn), multiple means of action and expression
to support strategic ways of learning (i.e., how we learn), and multiple means of engagement to support affective learning (i.e., why we learn). In addition, games provide the opportunity for learners to connect on a cultural level. In the following section, the authors further describe how mathematical games, when implemented strategically through an equitable lens, incorporate the three tenets of UDL: presentation, expression, and engagement.

**Analyzing Games Through the Universal Design for Learning Framework**

In order to overcome some of the obstacles students who struggle with mathematics face, classroom teachers need the pedagogical knowledge from teacher preparation programs and professional development to become cognizant of and gain experience with early interventions and effective strategies that may be implemented to provide all students with the greatest opportunity to learn. One support includes providing an explicit framework for curriculum analysis, lesson planning, and implementing instructional strategies based on the principles of UDL. This framework provides a specific lens through which teachers can deeply consider potential barriers that restrict equitable access to the mathematics in the lesson, and identify strategies to “support student engagement by presenting information in multiple ways, and allowing for students to access and express what they know in a variety of ways, [while also including] accommodations that should not alter the standards nor lower the expectations for students” (McNulty & Gloeckler, 2011, p. 6). The tenets of UDL provide valuable guidelines teachers can use in conjunction with their knowledge of student learning progressions and effective instructional strategies to use mathematical games as effective tools for engaging a diverse population of students in high quality mathematics.

Teachers need to be able to anticipate and identify potential barriers young students may experience when engaging in any mathematical task or activity. This applies to mathematical games as well. According to the principles of UDL, by anticipating potential barriers, classroom teachers may embed instructional strategies and scaffolds that support students’ needs; thus, providing opportunities that allow all students, including those who struggle, access to rigorous mathematics. In Table 1, provides an overview of potential barriers for each game discussed later in the chapter and its corresponding solution strategies. While the provided list is not exhaustive, the overview reveals how identifying such difficulties from the outset can direct classroom teachers to purposefully embed additional scaffolds or prompts that appropriately challenge students, while also including the necessary supports that provide access to the rigorous mathematics and problem solving tasks both at the forefront and throughout game play.

Furthermore, the UDL framework guides the discussion of possible solutions to address anticipated obstacles in each of the four focus games using elementary (age 5 to 12) students’ experiences, work samples, and conversations to explicitly describe and attend to:

1. Translating multiple solutions or strategies,
2. Accepting varied modes of expression, which allow for multiple entry points that stimulate rich mathematical discussions, and
3. Generating increased students’ interest and participation.

In the subsequent discussion the authors provide specific examples and dialogue from actual classroom episodes to provide a clear look into how the principles of UDL may be integrated into curriculum as a
lens through which teachers, both practicing and prospective, may analyze, modify, and apply pedagogical strategies focused on developing the mathematical knowledge and skills of all students.

**The Ants Go Marching: Measurement**

The game, The Ants Go Marching, may be used with a range of learners to reinforce measurement skills. In the activity, each student has an ant that he/she races from one end of the paper to the other. The ant moves according to the type of measurement unit selected (e.g., inch, centimeter) and the length of the measurement is determined by the roll of a die. To play the game, the teacher or student(s) first selects the measurement unit. Then, the students place their “ant” at one end of the paper. Each player rolls a die, and uses a provided manipulative (e.g., square inch tiles) to measure the traveled distance. Game play continues until one ant reaches the end of the paper.

**Presentation**

Students are presented with a measurement task through a game format, which often varies from traditional classroom measurement activities. Moreover, there are numerous alternate group formats (e.g., game played on the computer using virtual manipulatives). Students may use physical tools such as inch tiles, grid/graph paper, or rulers to provide different presentations of the selected measurement unit. Through the use of these tools, teachers provide scaffolds for students who may experience anticipated obstacles with measurement such as incorrectly iterating units.

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**Table 1. Potential barriers for the focus games**

<table>
<thead>
<tr>
<th>Game</th>
<th>Potential Barrier</th>
<th>Solution Strategy</th>
<th>Corresponding UDL Tenet</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Ants Go Marching</td>
<td>Tiles may slide during play causing measurement error</td>
<td>Grid paper, trace on edge of butcher paper, stop motion animation</td>
<td>Multiple modes of expression</td>
</tr>
<tr>
<td></td>
<td>Difficulties connecting numeral to quantity</td>
<td>Recreate dot pattern from die with blocks before placing blocks</td>
<td>Multiple modes of presentation</td>
</tr>
<tr>
<td></td>
<td>Students measure incorrectly such as using different units of measure or include gaps or overlaps</td>
<td>Create rulers and connect comparable non-standard units to rulers</td>
<td>Multiple modes of presentation</td>
</tr>
<tr>
<td></td>
<td>Activity is not challenging to students</td>
<td>Investigate various units and include prompts related to inverse relationship between size of unit and number needed to traverse length</td>
<td>Multiple modes of engagement</td>
</tr>
<tr>
<td>Wipeout</td>
<td>Students have difficulty pressing buttons on the calculator</td>
<td>Have calculators on iPads or large button calculators for children to use</td>
<td>Multiple modes of presentation</td>
</tr>
<tr>
<td></td>
<td>Students do not understand place value</td>
<td>Allow children to incorporate place value chart or Base 10 blocks as they explain their thinking with the calculator</td>
<td>Multiple modes of expression</td>
</tr>
<tr>
<td>Geoboard Battleshape</td>
<td>Students have fine motor difficulty with the Geoboard bands and pegs on the board</td>
<td>Use Geoboard paper for students to draw, which can be copied at different sizes</td>
<td>Multiple modes of expression</td>
</tr>
<tr>
<td>Roller Derby</td>
<td>Difficulty with numeral recognition</td>
<td>Develop Roller Derby board on a number line</td>
<td>Multiple modes of presentation</td>
</tr>
</tbody>
</table>
In the following excerpt from a conversation among kindergarteners (i.e., 5 and 6 year olds), it is apparent that the presentation of the game stimulated the student’s curiosity of the ruler. The classroom teacher addressed the students’ inquisitive nature by introducing the ruler (with both centimeter and inches) and provided probing questions to help guide the kindergarteners to make connections between the two representations of the length measurement.

Teacher: James, I heard you say something about a ruler. What is a ruler?
James: You use it to measure things.
Teacher: Are you talking about a tool like this [pulls out standard ruler with inches and centimeter measurements]?
James: Yes. See you put it like this and it tells you how many you go. See [puts ruler next to the inch tiles from the game and teacher adjusts so the ruler aligns to the start of the tiles].
Vinny: 30?
Teacher: What is 30?
Vinny: It says 30 on the top.
Teacher: What do you think the 30 means? James was telling us that this tool is used to measure. What do you think the 30 means if we are measuring?
Vinny: How far?
Teacher: What do you mean how far?
James: The ruler tells us how big something is. But it’s not that big. It’s not 30.
Teacher: What’s not 30?
James: This [points to the inch tiles]. We didn’t get to 30. We just count 11 on my book.
Teacher: So what did 11 tell you?
Vinny: That’s how many blocks our ant went.
Teacher: So did the 11 help us measure how far our ant walked in the game?
James: Yes. But, not to 30.

In this example, the students begin to make the connection between the ruler, the standard measurement tool, and the non-standard units of measure – the square tiles used in the game. Although the ruler was not used during game play and the conventions of the ruler were minimally discussed, the students had the opportunity to begin making connections between the different measurement tools. Therefore, the presentation of the measurement concept was conducted in two distinct formats.

In the summary portion of the lesson, the teacher led a guided discussion based on the discoveries of Vinny and James. Specifically, she acknowledged what the students found as they played the game and use guided prompts and questions to explicitly point out the relationship between the conventional ruler and the square tiles. Moreover, she addressed common misconceptions by reminding students that the tiles (i.e., units) should be placed end to end with no gaps or overlaps. Using purposeful errors of leaving a space between two of the tiles, she helped students see how the exact measurement was altered when these common mistakes occurred. For example, during the summary, the teacher re-measured the “distance of 11 tiles” and included additional space between the tiles. As a result, the teacher asked the question, “Does this still measure 11 tiles?” In the discussion, two students recalled events and expressed their understanding from their shared experience during the game.
Expression

Students express their understanding in multiple formats, such as engaging in discussions of noted patterns or translating their thoughts in math journals using words, numbers, or pictures. Teachers may facilitate discussions where the class creates new challenging rules or analyzes strategies for measurement and strategic game play.

As previously discussed, the summary component of the lesson—after the class had the opportunity to play the game and the teacher monitored strategies and solutions—allowed the students to reflect on their experience and express their understanding of the measurement concept. When the teacher prompted students with the question, “Does this still measure 11 tiles?” some students said yes because “it’s still 11 blocks.” However, one group responded “no” and were able to describe why they thought it would be different.

Charlie: No, no, no. It can’t be eleven. We were kind of doing extras in our game and I saw it was cheating.
Teacher: What do you mean?
Charlie: Well, we was putting our ant guy on the end of our blocks to measure. Then when we got a new number from the dice we put our blocks in front of our ant guy. But then I saw that when Angie moved her guy to the front again she cheated a little bit on accident because there was a big hole.
Teacher: Can you show us what you mean up on the document camera? [Teacher sets up document camera, blocks, and figure to represent the ant.]
Charlie: Yes. Can I have a dice please? [Puts out 3 blocks. Then puts figure for his “ant guy.” Then rolls again.] See 5. So I put 5 blocks in front of my guy. 1, 2, 3, 4, 5. Then I move my guy. But when I watched Angie do it there was an extra space when she moved the guy. See right here [points to the space between the 3 initial blocks and the 5 new blocks added to the length measurement]. You can put another block here. So, really it’s not the same. It’s one more.
Teacher: So, what does that tell us about the spaces in between? Is it the same measurement?
Randy: It’s the same number of blocks—still eleven.
Teacher: Yes, you’re right that it is the same number of blocks. But we’re now thinking about the distance our ant traveled. Did our ant go the same distance if we spread the blocks out?
Randy: No.
Teacher: So, when we measure we need to make sure that our tools are lined up right next to each other so that no other ones can fit in between them.
Charlie: Yeah, cause if you skip a space like we did it’s kind of cheating because it’s not the same number.

In this conversation, students were allowed to express their ideas and explore the misconceptions through experiences they had in the game. Moreover, students were encouraged to represent their thinking by recreating the game scenario and connect the idea to the question posed by the teacher. Following up the discussion with independent reflections in math journals using words, numbers, or pictures may provide an additional opportunity for students to consider the question and the responses of his or her peers. The teacher’s question built upon a common measurement misconception expressed by students and stimulated students’ further thinking about the key concept. In subsequent activities and discussions students can continue to explore the critical idea of measurement with regard to iterating a unit with no gaps and overlaps. Moreover, common misconceptions such as utilizing different units of measure rather than iterating the same unit over the measured distance, may arise. 
While the main focus of the lesson surrounds the concept of measurement, it is important to recognize that there may be additional barriers in the lesson related to constructs such as counting. For example, students may have difficulties connecting numeral to quantity. By anticipating this potential barrier using the principles of UDL, students who experience this difficulty could be supported by recreating the dot patterns on the die with the blocks before placing the blocks to represent the distance his or her ant would move on that turn.

Engagement

The game promotes appropriate modifications that stimulate higher order thinking for a diverse population of students. The game format may be readily altered to coincide with specific themes such as Fall, football, and school or community events. Furthermore, the game is flexible so modifications may be embedded in the game to appropriately challenge students who already demonstrate the foundational measurement and computational knowledge and skills. Teachers can incorporate multiple dice or encourage students to explore the inverse relationship between the size of the unit and the number of iterations to traverse a distance or space. Furthermore, students may use yarn to trace straight or curved paths to investigate the difference between the distance traveled.

While the game offers several possibilities for engaging students in relevant or meaningful contexts, stimulating mathematical thinking at appropriate experience levels, or extending the concept to developing strategies for measuring curved paths, students—even those as young as kindergarten—generate questions they want to investigate as they play the game. By stimulating further inquiries, the game serves as a catalyst for students to transfer the mathematical idea to novel contexts; thus potentially deepening the learner’s understanding of the idea. For example, as kindergarten students played *The Ants Go Marching*, one child remarked to another, “What if an elephant walked this far? It would only take 3 steps and the ant would take 50 hundred steps.” While this statement may initially seem innocent, the comment actually implies the child is making connections of the inverse relationship between the size of the unit and the number of iterations needed to measure an attribute such as length. Furthermore, capitalizing on this prediction and engaging students in a subsequent exploration to follow up the statement not only values the child’s thinking, but also promotes continued interest in a critical concept in early mathematics.

**Wipe Out: Number Sense and Base 10**

Wipe out is a calculator game that emphasizes key concepts related to place value. Students begin by entering a number on a calculator with a predetermined number of digits. Initially each digit in the number must be different (e.g., 333 would not be permitted) and greater than zero. However, as students advance in the game some rules can be altered and the number may include repeated digits (e.g., 3,239 could be used).

The goal of the game is to Wipe Out the entire number, one digit at a time, by performing an operation that would result in particular place values reflecting zero. The game begins after each student has recorded the “target number” with a predetermined number of digits. For example, if the required number of digits is five the number 68459 could be used. Then a selected individual spins a 1 – 9 spinner and asks his/her peers to Wipe Out that digit. Students must attend to the place value of the digit in order to correctly input the operation and quantity. For instance, if the teacher calls out the number 5, the student
should subtract 50 in order to Wipe Out the 5 and have that place value reflect zero. The first person to Wipe Out their number to zero is the winner.

Presentation

Wipe Out may be played with or without a calculator. Using both methods provides an opportunity for students to explore the concept of place value and reflect on the conventions of the base-10 system. For instance, when students play the game without the calculator many will use the U.S. standard algorithm, or the column method, for subtraction and line up the digit on the spinner in the correct place value column many times without including the appropriate place holder zeros. While in the written version of the game, this mode of representation does not necessarily impede game play, when students transfer the game to the calculator mode the absence of the place holder zeros in the tens place or greater substantially impacts progress. Thus, requiring students to play the game with the calculator as another representational mode supports students’ development of place value understanding to focus the students’ attention on the mathematics involved. Moreover, by anticipating these difficulties and looking for how students attend to place value provides teachers with opportunities to discuss common misconceptions.

Expression

Throughout the game the students have an opportunity to use precise language and mathematical terminology to describe their strategies for wiping out designated numbers. In the following excerpt from a group of third graders playing the game, the multiple means of expression that students used to communicate their thinking encouraged rich mathematical discussions and allowed for multiple entry points so that students with varying levels of understanding could engage in not only the activity, but also the subsequent discussion.

Teacher: I think I heard someone say Wipe Out. Who has their number wiped out?
Marshall: Me! I wiped out my number.
Teacher: Tell us about how you—
John: No! I wiped out mine, too! I have just zero left!
Teacher: Okay, John. Let’s have Marshall tell us about his number and then you can tell us how you winked out your number. Marshall, talk about what number you started with and how you wiped out to zero.
Marshall: I picked the number seven two four six.
Teacher: What number is that? You told me a bunch of digits by themselves, but what number do we call that? How do we say that number?
Marshall (and some other voices): Seven thousand two hundred forty-six.
Teacher: Marshall, how do you know that number is seven thousand two hundred forty-six?
Marshall: I used my columns. I put the chart at the top of my paper and then I put the number I wanted in the columns. Then I colored the ones and tens and hundreds to remember those go together and then the other one was the thousands.
Teacher: How did you think about how to put those columns together when you read it?
Marshall: Because the thousands just had 7 so I know that’s seven thousand. Then I remembered that I read the ones and tens and hundreds together just like normal numbers. So there was a two and a four and then a six so that means it was two hundred forty-six.

John: I had the same numbers, but I made a different number with them. I picked a four for my thousands place and then I had the other ones. But then I wanted to make the biggest number so I made them seven thousand and then six hundred forty-two. So my numbers went seven, six, four, two. I made the flats and the rods and the dots and the cubes on my page on top of the numbers so I would remember them. (Teacher records the place value columns with the hundreds-tens-ones period all the same color and draws the base-10 block representation above each column. Then she writes each number below the values in the chart).

Madeline: [Teacher], I kept messing up in the first one because I forgot my zeros. But then I figured out a thing to do that would help me remember.

Teacher: How did you use your strategy, Madeline? Can you tell us what you did to help you think about the important place holder zeros?

Madeline: I made mine into an addition sentence. I didn’t use the same numbers that they did, but I did my number like 5,000 + 200 + 90 + 4. In the first game, when you spun the numbers I would just do the minus with that number and I kept getting crazy numbers that didn’t wipe out. So, then I saw somebody else do the addition sentences, and so I tried it. It worked. Then when you spun the 2 I remembered to put 200 in my calculator and not 2. But, it was funny because when you spunned [sic] the 4, I could still just do 4 because it was just ones.

In this excerpt from a third grade class discussion, the students were able to demonstrate their understanding through multiple modes of expression including place value charts, base-10 block references, and expanded form of multi-digit numbers. By allowing for multiple means of action and expression the classroom teacher supports his or her students’ strategic ways of learning.

Engagement

During the game, students can engage in collaborative learning—playing the game with large or small groups. However, students can also play independently either to practice the game as indicated, or they can explore the frequency of the spins for each digit. With the game directions being minimal and the game objectives being clear (i.e., use place value knowledge to wipe out your digits so you end up with zero) several students were even more engaged adapting the game with their own “improvements.” The following vignette features a conversation between two groups of students describing the different games they “made up” during the exploration component of the math workshop.

Teacher: So, what have you guys been talking about? I haven’t seen much of the Wipe Out game over here.
Tara: That’s because we’re making a new game. Ours is still like Wipe Out, but not really the same. But Sam and Lennox made up a word game with theirs.
Teacher: Ooooh. A word game, huh?
Sam: Yeah. But, there’s two ways to play it.
Teacher: Okay Lennox, why don’t you tell me about your game? How do you play it?
Lennox: So, me and Sam found words we can make on our calculators. Like I did Bob. And then the game is I say Bob. And then Sam has to figure out how to make that word on his calculator, and then tell me what number I made. See, so Bob is eight zero eight. So, Sam had to tell me—
Sam: Eight hundred and eight.
Teacher: Eight hundred and eight what? Oh, you mean eight hundred eight?
Sam: Yes.
Rylen: And then Sam made the word go and Lennox put 90 in her calculator because the 9 was like a g and the 0 was like an o so it made go.
Sam: But now we can’t think of any more words.
Teacher: Well, one thing I noticed is that when you were showing me things, I was looking at your calculator upside down. So the numbers look different upside down. I wonder if you can make more words if you put numbers in your calculator and then look at the calculator upside down. I think your game is neat!

In these scenarios the students were motivated to learn, which supported their affective development, or as the National Research Council (2001) discussed in their seminal work, *Adding It Up,* the students’ productive disposition was stimulated. Games like Wipe Out that offer opportunities for exploration, extension, and transfer of concepts can reduce math anxiety, enhance students’ inclination to view mathematics as having a purpose and being relevant and meaningful, and encourage perseverance.

**Geoboard Battleshape: Geometry**

Geoboard Battleshape reflects the traditional Hasbro game, Battleship©. In the geometry version, the goal is to sink the opposing team’s Battleshape using knowledge of the coordinate system. While the game can be played in any—or all—of the four quadrants of the Cartesian plane, in elementary grades, the game is typically played in the first quadrant. To play, each one or two-player team has five shapes (e.g., triangle, square, pentagon, trapezoid, hexagon). The team secretly places the shapes on his or her geoboard. After all Battleshapes are placed, the opposing team attempts to “sink” the Battleshapes by calling out coordinates that “hit” the vertices of the opposing team’s shapes. Once all vertices have been “hit,” the ship is not sunk until the Battleshape is correctly, and precisely, named.

**Presentation**

Geoboard Battleshape may be played in many different formats to reinforce the concept of the coordinate plane and review shape names and characteristics. For example, students may represent the shapes on:

1. A Geoboard grid,
2. Geoboard paper, or
3. A life-size coordinate plane taped to the floor.

These multiple means of representations may support how students associate meaning to the concepts and direct interpretations toward the conventions of the Cartesian plane and the defining attributes of the shapes. Moreover, the critical vocabulary related to the geometric shapes or the Cartesian plane may be discussed in various formats through the game. While activating students’ prior knowledge in relation
to their lived experiences is important during the introduction to the lesson, reiterating key terms such as plane or horizontal on the grid systems during game play can assist in illustrating the concepts and increase comprehension.

Expression

The conventions of the Cartesian system can be difficult for students to recall (Battista, 2007). Therefore, encouraging students to express their thinking supports strategic ways of learning that can be shared and compared as students play the game. Students can develop various ways to help them remember the coordinate sequence or to begin finding coordinates from the origin. These expressions not only highlight how the students learn and remember, but includes various formats through which students can demonstrate personally connected mnemonic creations in art, poetry, or multi-media. For instance, second grade students can share posters they have created with brief “memory strategies” such as “First, I go OVER to my friend’s house, and then I go UPstairs to play,” or “I run OVER before I can jump UP.”

The conglomeration of various forms of expression supports all students and caters to unique ways in which they prefer to demonstrate their knowledge, while also offering a means to compare and evaluate different representations of the same mathematical concept in rich mathematical discussions. This process provides an appropriate challenge for all students because it allows for multiple entry points and engages individual interest both during and after the game.

Engagement

Because of the familiarity of the game in most communities, family members can reinforce key concepts at home as they play a new version of the classic. Furthermore, the game can be adapted to the community by altering the focus from geometric shapes to focus solely on coordinate systems and the Cartesian graph. Consequently, students may make connections between Social Studies standards related to geography or community by developing local maps and plotting different buildings or locations in the model of the area.

Roller Derby: Data Analysis/Probability

To play Roller Derby, students use knowledge of basic facts, decomposing numbers, and probability to compete in a game of chance involving the sums of the numbers on the dice. The game begins after each player places 12 markers into the columns of the game board (with columns 1 – 12) in any way he or she chooses. Once all players have distributed their markers, the players take turns rolling a pair of dice (youngest player rolls first). When the dice are rolled all players in the game find the sum of the numbers on the dice. If a player has a marker in the column that corresponds to the sum, he or she removes one marker from the game board. If that column is blank, the player cannot remove any markers. The first person to remove all of his or her markers wins.

Presentation

Although Roller Derby addresses content standards related to Data Analysis and Probability, one of its key constructs is finding sums of two single digit numbers. Young students may struggle with devel-
oping or applying strategies to compute the sums. Offering double ten frames and two color counters provides students another mode to present the mathematical idea; thus supporting all students. Moreover, to encourage the transition from solely focusing on the computation side of Roller Derby, classes can track the expressions rolled on the dice and record them—once the expression is recorded it cannot be recorded again—in the various Derby columns. The various records of generated data can help students recognize that the sample space can be represented in multiple ways (e.g. addition table, line plot, list).

As students play more rounds of the game they gather additional evidence that should generate questions and hypotheses as students analyze the data and teachers scaffold probability language such as “impossible,” “most likely,” “least likely,” and “equally likely.” Through this presentation, idiosyncratic student representations, students recognize both the computation content and the data analysis concepts to assign meaning to these big mathematical ideas. Furthermore, additional game play contributes to students not only using different computation strategies, but also integrating various methods to arrange the counters in attempts to strategically win the Roller Derby championship.

Expression

As students participate in the Roller Derby challenge, there is a foundational construct through which students can express strategic ways of learning and demonstrate mathematical understanding—computation strategies toward developing fact fluency. The repetitive nature of the dice rolling provides ample opportunities for students to practice or reinforce their basic facts. As they are exposed to the same set of problems over and over, students will begin to notice patterns and relationships. Moreover, they may become inclined to look for more efficient ways to solve problems. From the following exchange between two first grade students, it can be argued that the game not only reinforces strategies to build fluency through repeated exposure to related problems in a set, but also that the social interactions during the game allow for opportunities to demonstrate understanding in multiple ways through the negotiation of meaning.

Myra: Hey! It’s 5 again. Five and four.
Hank: I like fives. I can do them fast.
Myra: Me too. I just put up my five and then I can count four more. I’m really fast.
Hank: No. You don’t have to.
Myra: What?
Hank: You don’t have to count.
Myra: Yes, you do. It’s a plus. Plus means you count bigger.
Hank: No. You don’t have to count. You just do five [holds up five fingers on one hand] and then you do four [puts up four fingers on the other hand] and then you can just see it’s nine ‘cause one more [wiggles thumb] is ten.
Myra: I know. That’s what I said.
Hank: No. You said you counted.
[Teacher calls out 3 and 4]
Myra: Three and four! I got this number, too, I think [quickly puts up group of three fingers on one hand and four on the other hand, then counts all fingers one by one].
Hank: See you counted.
Myra: No. I knowed [sic] it. I just was checking.
Hank: No. You counted. You know it. When [teacher] said it. I didn’t have to count because I had my fingers up still. I just put my one finger down ’cause I did four.

Myra: I was still right. Did you get to take off “7”? I did.

The debate between the two first graders exemplified how they generated arguments to support their thinking and reasoning, and these justifications were further enhanced by the students’ use of their fingers as manipulatives. Here, both students were able to engage in the game, whether they counted all to solve or were using more flexible number sense and numerical relations to find the sum of the two addends. Either way, the students were each able to participate in the mathematical conversation by expressing their thinking, while also feeling success in their proficiency in solving the given problems.

Engagement

The Roller Derby challenge encourages students to remain engaged as they work to identify patterns that can help them determine the most effective way to distribute their tiles. Providing students opportunities to hypothesize the “best” column(s) to place their tiles, test their prediction, and then alter their approach as they gain new information supports both a systematic and creative approach to the structure of the game. Furthermore, the game provides an additional forum to explore early algebra through discoveries in generalizations and patterns, and students have the potential to apply co-variation and compensation strategies as they experience related problems with a limited set of addends. In addition, students have an opportunity to begin investigating the principles of probability; noting that although theoretical probability exists, it does not always reflect the experimental results.

FUTURE RESEARCH DIRECTIONS

Research suggests that teacher preparation programs should provide new teachers with the opportunity to master initial attitudes, skills, and knowledge required to be successful as beginning teachers in inclusive classrooms (Loreman, 2010). Walton, Nel, Muller, and Lebeloane (2014) further support the idea of preparing prospective teachers for inclusive teaching in their preparation program. Although teacher education is a vital component of teacher preparation (e.g., Forlin, 2010), discussions related to improving the preparation of teachers have been framed broadly in teacher education, but also within specific disciplines such as mathematics education (Conference Board of the Mathematical Sciences, 2001; 2012).

To address the need to better prepare teachers, it is imperative that a critical evaluation be done to analyze programs for teacher preparation. However, a useful knowledge base for mathematics teacher education is lacking, even with years of research on mathematics teaching in grades K-12. For example, there is no shared professional curriculum for preparing prospective teachers of mathematics (Ball, Sleep, Boerst, & Bass. 2009; Zaslavsky, 2007). Moreover, the mathematics education research community knows very little about the practices of faculty who prepare teachers of mathematics (i.e., mathematics teacher educators), as these practices are not widely documented or disseminated (Bergsten & Grevholm, 2008; Even, 2008; Floden & Philipp, 2003; Hiebert, Morris, & Glass, 2003; McDuffie, Drake, & Herbel-Eisenmann, 2008; Schempp, 1995). Therefore, the next steps the authors propose are to generate evidence from mathematics teacher educators about effective instruction and pedagogical actions at the collegiate level in relation to using the UDL framework in teacher preparation programs.
However, more empirical evidence is needed regarding several outliers variables. Specifically, future studies that explore:

1. How UDL is—or is not—addressed in teacher education and professional development, and
2. The questioning teachers use within the tenets of UDL that provide additional opportunities for students by helping them connect representations, clarify means of expression, and extend opportunities for further engagement and transfer of knowledge to novel contexts.

CONCLUSION

Equity does not imply identical instruction, but focuses on instruction that includes appropriate accommodations, which provide opportunities for students to learn and be engaged in rigorous mathematics (NCTM, 2000). Therefore, it is crucial that researchers, mathematics teacher educators, and school district personnel recognize and act upon the need for explicit teacher preparation and professional development that incorporates the UDL framework, which may be used as a tool to more effectively instruct students who struggle with mathematics by providing strategies and practices that specifically address their needs.

Universal Design for Learning is an equitable approach to instruction that teachers may use to analyze mathematical tasks and subsequently include appropriate accommodations that meet the needs of students with a broad range of skills. Universal Design for Learning works under the premise that all lessons contain potential obstacles for students. Teachers who incorporate the key ideas of UDL in their planning by anticipating barriers and embedding effective strategies within the lesson that are intended to scaffold learners against these obstacles provide access and opportunity to all students in the classroom without compromising high quality mathematics instruction. Thus, UDL provides both practicing and prospective teachers with a valuable framework to analyze mathematical tasks, such as games, and effectively consider differentiation options that appease the needs of a diverse population of students. By disseminating specific applications of the preceding content in the context of teacher education and professional development, the authors believe this information will support the work of mathematics teacher educators and researchers in the field.

REFERENCES


### Defining Effective Learning Tasks for All


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Walton, E., Nel, N. M., Muller, H., & Lebeloane, O. (2014). ‘You can train us until we are blue in our faces, we are still going to struggle’: Teacher professional learning in a full-service school. *Education as Change, 18*(2), 319–333. doi:10.1080/16823206.2014.926827