## GENERAL

- Atmosphere is very thin compared to size of earth.
o If earth were a basketball, our atmosphere would be as thick as a sheet of paper.
o No defined upper limit to the atmosphere
- Energy source for the atmosphere is the Sun
o The Earth's interior heat is a negligible heat source for the atmosphere.


## COMPOSITION

- Since air is a mixture of ideal gases, a volume percentage of its components is the same as a molecular percentage.
- Air is composed of fixed gases, variable gases, and aerosols
- Permanent (constant percentage) gases
- Nitrogen ( $\mathbf{N}_{2}$ )
- Oxygen $\left(\mathrm{O}_{2}\right) \quad-21 \%$
- Argon (Ar) - 0.9\%
- Neon (Ne)
- Helium (He)
- Methane ( $\mathrm{CH}_{4}$ )
- Krypton (Kr)
- Hydrogen $\left(\mathbf{H}_{2}\right)$
- Variable gases
- Water vapor $\left(\mathrm{H}_{2} \mathrm{O}\right)$
- Carbon dioxide ( $\mathrm{CO}_{2}$ )
- Ozone (O3)
- Aerosols


## EVOLUTION OF ATMOSPHERIC OXYGEN

- The Earth was formed around 4.5 billion years ago.
- The very first atmosphere likely consisted of
o Hydrogen (H)
o Helium (He)
o Methane ( $\mathrm{CH}_{4}$ )
o Ammonia ( $\mathbf{N H}_{3}$ )
- This first atmosphere was lost to space during the bombardment by meteorites and the intense solar wind shortly after the Earth was formed.
- A new atmosphere eventually formed from outgassing from volcanoes and other venting activity.
o This second atmosphere was composed primarily of
- Water Vapor ( $\mathbf{H}_{2} \mathrm{O}$ )
- carbon dioxide $\left(\mathrm{CO}_{2}\right)$
- nitrogen ( $\mathbf{N}_{2}$ )
o There were only trace amounts of oxygen in this first atmosphere.
- This oxygen likely came from the reaction of ultraviolet radiation with water, which splits the water into hydrogen and oxygen.
o $\mathbf{O}_{2}$ was less than 2 ppmv (parts per million by volume), or about $\mathbf{1 0 0 , 0 0 0}$ times less than today.
- Photosynthesis evolved sometime between 3.5 and 2.7 billion years ago.
o Photosynthesis evolved first in bacteria, called cyanobacteria.
o Photosynthesis takes in carbon dioxide $\left(\mathrm{CO}_{2}\right)$ and gives off oxygen $\left(\mathrm{O}_{2}\right)$.
- Rather than build up in the atmosphere, the oxygen formed from photosynthesis would have instead reacted with other substances such as iron (Fe) to form iron oxides in the rocks and soils.
- Eventually though, all the oxidation was essentially complete, and then oxygen could build up in the atmosphere.
- Between 2.4 and 2.0 billion years ago oxygen levels rose to somewhere between $1 / 10$ or more of their present value, and then remained constant until around 850 million years ago.
- Oxygen levels began to rise around 850 million years ago, and 300 million years or so ago they were actually higher than today (perhaps as much as $35 \%$ of air molecules were oxygen).
o This oxygen spike during the Carboniferous period is correlated to the finding of fossils of giant insects and amphibians.
- Certain insects and amphibians rely on diffusion of oxygen for respiration, and oxygen can diffuse farther at higher concentrations.
- Some dragonflies had 30 inch wingspans, and had bodies over 1 inch in diameter!
- Oxygen levels fell to present levels ( $21 \%$ ) by about 200 million years or so ago, which ended the era of giant insects and amphibians.
- During the evolution of the atmosphere, the amount of nitrogen, $\mathrm{N}_{2}$, remained relatively constant.


## IDEAL GAS LAW

- Air behaves like an ideal gas
- An ideal gas has the following conceptual properties
o Forces between molecules are ignored. No interactions except during collisions.
o Molecules are assumed to be point masses (have no volume themselves)
o Collisions are elastic.
- In an ideal gas, volume depends only on pressure, temperature, and number (not the type) of molecules.
o One cubic meter of hydrogen contains the same number of molecules as one cubic meter of carbon dioxide, if both gasses are at the same temperature and pressure.
- The ideal gas law (also known as the equation of state) relates pressure, temperature, and density (or volume).
o In chemistry the ideal gas law usually appears as

$$
\begin{equation*}
p V=n R T \tag{1}
\end{equation*}
$$

where $\boldsymbol{p}$ is pressure, $\boldsymbol{V}$ is volume, $\boldsymbol{n}$ is the number of moles of gas, $R$ is the universal gas constant, and $T$ is absolute temperature.
o In meteorology we use a different form, which is derived by first dividing both sides of (1) by volume, and then multiplying top and bottom of the right-hand side by molar mass (molecular weight), $M$, to get

$$
\begin{equation*}
p=\frac{n}{V} R T=\frac{M n}{V} \frac{R}{M} T . \tag{2}
\end{equation*}
$$

Recognizing that $M n$ is the mass of the gas, so that $M n / V$ is the density, we end up with

$$
\begin{equation*}
p=\rho \frac{R}{M} T . \tag{3}
\end{equation*}
$$

We then define $R / M$ to be a new constant, $R^{\prime}$, called the specific gas constant, so that the ideal gas law is now

$$
\begin{equation*}
p=\rho R^{\prime} T \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
R^{\prime} \equiv R / M . \tag{5}
\end{equation*}
$$

o IMPORTANT: The specific gas constant is not really a constant! It is different for different gasses. When using the ideal gas law, you must use the specific gas constant for the gas you are working with.

- For dry air, the specific gas constant is called $R_{d}$, and has a value of $287.1 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{\mathbf{- 1}}$.

$$
\begin{equation*}
p=\rho R_{d} T \tag{6}
\end{equation*}
$$

The reason (6) is more popular than (1) with meteorologists is that in the atmosphere it makes more sense to think more in terms of air density, rather than in the volume and number of moles of the air.

## PRESSURE AND DENSITY

- Density is mass per volume
o Density is determined by the mass, and number, of molecules.
o Air is compressible, which means that you can squeeze the molecules closer together and increase density.
o Air at the surface of the earth is compressed more than air at the top of the atmosphere (because of the weight of the air above it).
+ Therefore, density is greatest at the surface of the earth, and decreases as you go up.
- Pressure is force per area
- There are two types of pressure
o Hydrostatic pressure, which is just due to the weight of the air above you
o Dynamic pressure, which is due to the motion of the air
- In most meteorological applications dynamic pressure is small compared to hydrostatic pressure, and we will assume for now that atmospheric pressure is solely due to hydrostatic pressure.


## HYDROSTATIC BALANCE

- In an atmosphere at rest there are three forces acting on an air parcel.
o Gravity acting downward
o Pressure force acting upward
o Pressure force acting downward
- The picture below shows a cylindrical air parcel of mass $m$ and with top and bottom surface area $A$, with these forces acting on it.
o $\boldsymbol{p}_{z+\Delta z}$ is the pressure at the top of the air parcel, and $\boldsymbol{p}_{z}$ is the pressure at the bottom.

- If the air parcel is at rest, then from Newton's $\mathbf{2}^{\text {nd }}$ law of motion the forces must sum to zero,

$$
\begin{equation*}
A p_{z}-A p_{z+\Delta z}-m g=0 \tag{7}
\end{equation*}
$$

The mass of the air parcel is given by its density multiplied by the volume, or

$$
\begin{equation*}
m=\rho A \Delta z \tag{8}
\end{equation*}
$$

If we put (8) into (7) and rearrange, we get

$$
\begin{equation*}
\frac{\left(p_{z+\Delta z}-p_{z}\right)}{\Delta z}=-\rho g . \tag{9}
\end{equation*}
$$

As the air parcel gets infinitesimally small, we take the limit of this as $\Delta z \rightarrow 0$ to get

$$
\begin{equation*}
\lim _{\Delta z \rightarrow 0} \frac{\left(p_{z+\Delta z}-p_{z}\right)}{\Delta z}=\frac{d p}{d z} \tag{10}
\end{equation*}
$$

(this is just the definition of a derivative from Calculus!)

- So, for an atmosphere at rest be now have

$$
\begin{equation*}
\frac{d p}{d z}=-\rho g \cdot \mathbf{1}^{\mathbf{1}} \quad \text { Hydrostatic Equation } \tag{11}
\end{equation*}
$$

- Equation (11) is known as the hydrostatic equation.
- The difference between the upward and downward pressure forces is known as the pressure gradient force.
- In hydrostatic balance, the vertical pressure gradient force is exactly balanced by gravity.
- Notice that in a hydrostatic atmosphere, $d p / d z$ is negative. This makes sense, since pressure decreases as you go upward (increasing $z$ ).


## HOW PRESSURE CHANGES WITH HEIGHT

- We can integrate the hydrostatic equation to find out how pressure changes with height.
- Integrating the hydrostatic equation would give an equation for how pressure changes with height. If we do this, we get

$$
\begin{equation*}
p(z)=p_{0} \exp \left(-\int_{0}^{z} \frac{g}{R_{d}} \frac{d z}{T}\right) \tag{12}
\end{equation*}
$$

O However, this integral is not straightforward for a real atmosphere. For an isothermal atmosphere ( $T$ constant with height), however, (12) becomes

$$
\begin{equation*}
p(z)=p_{0} \exp \left(-\frac{g}{R_{d} T} z\right) \tag{13}
\end{equation*}
$$

where $p_{0}$ is the pressure at the surface.
○ Defining the scale height as

[^0]\[

$$
\begin{equation*}
H=\frac{R_{d} T}{g} \tag{14}
\end{equation*}
$$

\]

we get an equation for pressure versus height in an isothermal atmosphere as

$$
\begin{equation*}
p(z)=p_{0} \exp (-z / H) \tag{15}
\end{equation*}
$$

- The figure below shows a plot of pressure versus altitude $(z)$ for three values of scale height. The scale height is the height at which the pressure is $\mathrm{e}^{-1}(\sim 37 \%)$ of the surface value, also referred to as the e-folding distance. The scale height gives an indication of how rapidly pressure decreases with height. A smaller scale height means a quicker decrease with height.

- Thought the actual atmosphere isn't isothermal, the equation above does a pretty good job for ballpark figures. The typical scale height used is $\boldsymbol{H} \boldsymbol{\approx} \mathbf{8 . 1} \mathbf{~ k m}$ (which corresponds to an average temperature of about 277 K ).
- A rule of thumb...the pressure drops by $1 / 2$ for every $5.6 \mathbf{k m}$ of altitude.
- Since density is related to pressure via the ideal gas law, we find that density in an isothermal atmosphere also decreases exponentially with height according to

$$
\begin{equation*}
\rho(z)=\rho \exp (-z / H) \tag{16}
\end{equation*}
$$

- A more realistic pressure profile is obtained if instead we allow temperature to decrease linearly with height according to

$$
\begin{equation*}
T(z)=T_{0}-\gamma z \tag{17}
\end{equation*}
$$

where $\gamma$ is the typical tropospheric lapse rate of $6.5^{\circ} \mathrm{C} / \mathrm{km}$. In this case equation (12) becomes

$$
\begin{equation*}
p(z)=p_{0}\left(\frac{T_{0}-\gamma z}{T_{0}}\right)^{\frac{g}{\gamma R_{d}}} . \tag{18}
\end{equation*}
$$

- A plot of equation (18) (dashed line) compared to (16) (solid line) is shown below. Note that the differences are not tremendous, further giving confidence to the use of the more simplified equation for back-of-the-envelope calculations.


LAPSE RATE

- The lapse rate indicates how rapidly the temperature decreases with height.

Mathematically, it is defined as

$$
\begin{equation*}
\gamma \equiv-\frac{d T}{d z} \quad \text { Definition of Lapse Rate } \tag{19}
\end{equation*}
$$

o Lapse rate is defined with a negative sign. Thus, if temperature decreases with height the lapse rate is positive.

- CAUTION: When converting the units of a lapse rate from one temperature scale to another, remember that we are dealing with temperature changes.

Thus, you have to remember the following:

o A change of $1^{\circ} \mathrm{C}$ is equivalent to a change of $1.8^{\circ} \mathrm{F}$ (not $33.8^{\circ} \mathrm{F}$ !)
o This is a commonly made mistake.
o As an example consider the following problem:
The lapse rate is $4.5^{\circ} \mathrm{C} / \mathrm{km}$. What is this in $\mathrm{K} / \mathrm{km}$ and ${ }^{\circ} \mathrm{F} / \mathrm{km}$ ?
Answer:

$$
\begin{aligned}
& \frac{4.5^{\circ} \mathrm{C}}{\mathrm{~km}} \bullet \frac{1 \mathrm{~K}}{{ }^{\circ} \mathrm{C}}=\frac{4.5 \mathrm{~K}}{\mathrm{~km}} \\
& \frac{4.5^{\circ} \mathrm{C}}{\mathrm{~km}} \bullet \frac{1.8^{\circ} \mathrm{F}}{{ }^{\circ} \mathrm{C}}=\frac{8.1^{\circ} \mathrm{F}}{\mathrm{~km}}
\end{aligned}
$$

## THERMAL STRUCTURE OF THE ATMOSPHERE

- The atmosphere can be divided into different layers based on its thermal (temperature) structure. These layers are differentiated by whether the lapse rate is positive or negative.
- The layers, from bottom to top, are:

| Layer | Lapse <br> rate | Mean <br> Altitude <br> $0-11 \mathrm{~km}$ | Remarks <br> - Contains majority of atmosphere. <br> - Where most "weather" occurs. |
| :--- | :--- | :--- | :--- |
| Troposphere | + | - Temperature decreases with height because heat source <br> is bat botom (due to Sun's rays striking earth. |  |
| Stratosphere | - | $11-47 \mathrm{~km}$ | - Thickness (height) varies with season and location. <br> Higher in summer and in Tropics. <br> - Contains ozone layer. |
| - Temperature increases with height due to absorption of |  |  |  |
| UV rays by ozone. |  |  |  |

Mesosphere $+\quad 47-85 \mathrm{~km}$
Thermosphere - $\quad>85 \mathrm{~km} \quad$ - Temperature increases because heat source is at top (due to absorption of Sun's rays by molecular nitrogen and oxygen).

- Temperature is hot, but it wouldn't feel hot. This is because there are so few molecules.
- The levels separating the layers are named by taking the prefix of the layer below and putting the suffix -pause with it.
o Example - The top of the troposphere is the tropopause.


## HETEROSPHERE AND HOMOSPHERE

- Lower part of atmosphere (below about 80 km ) is well mixed (fixed gases are found in constant proportions).
o The well-mixed layer is called the homosphere or also the turbosphere.
o Above the turbosphere is found the heterosphere, which is not well mixed. Lighter molecules found at higher altitudes.


## IONOSPHERE

- The ionosphere is the region above about 60 km , where there are numerous ions and free electrons present.
o Begins in the mesosphere and extends upward through troposphere.
o Important for HF and AM radio propagation.


## GOVERNING EQUATIONS

- The dynamics and motion of the atmosphere are governed by a system of seven equation and seven unknowns.
- These equations are known as the primitive equations or the governing equations.
- These equations are:
- Three momentum equations, which express Newton's Second Law for an air parcel in each of the three spatial directions.

○ The mass continuity equation, which expresses the conservation of mass of the atmosphere.

O The water-mass continuity equation, which expresses the conservation of water mass in the atmosphere.

O The thermodynamic energy equation, which expresses the conservation of thermal energy.

○ The ideal gas law, which relates the thermodynamic variables.

- The unknowns for these equations are:

O The wind speeds in each of the three spatial directions.
○ Temperature
O Pressure
O Density
O Humidity

## EXERCISES

1. From the ideal gas law $p V=n R T$, calculate how many molecules are contained in a cubic centimeter $\left(\mathrm{cm}^{3}\right)$ of air at a pressure of 1013.25 mb and a temperature of $15{ }^{\circ} \mathrm{C}$ ? ( $R=8.3145{\left.\mathrm{~J}-\mathrm{mol}^{-1}-\mathrm{K}^{-1} ; N_{A}=6.022 \times 10^{23} \text { molecules } / \mathrm{mol}\right) ~}_{\text {m }}$
2. How many oxygen molecules are there in $\mathrm{cm}^{\mathbf{3}}$ of air at a pressure of $\mathbf{1 0 1 3 . 2 5} \mathbf{~ m b}$ and a temperature of $15^{\circ} \mathrm{C}$ ?
3. The table below gives the molecular weights and volume percentages for the standard atmosphere. Use them to show that the molecular weight of air is $28.964 \mathrm{~g} / \mathrm{mol}$. (Hint: Start with 1 mole of air. Figure out how many moles of each gas there are, and then find their weights. Then add the weights.)

| Gas | $\mathrm{M}(\mathrm{g} / \mathrm{mol})$ | \% by volume |
| :--- | :--- | :--- |
| $\mathrm{N}_{2}$ | $\mathbf{2 8 . 0 1 3 4}$ | $\mathbf{7 8 . 0 8 4}$ |
| $\mathrm{O}_{2}$ | 31.9988 | $\mathbf{2 0 . 9 4 7 6}$ |
| Ar | $\mathbf{3 9 . 9 4 8}$ | $\mathbf{0 . 9 3 4}$ |
| $\mathrm{CO}_{2}$ | $\mathbf{4 4 . 0 0 9 9 5}$ | $\mathbf{0 . 0 3 1 4}$ |

4. Show that the specific gas constant for dry air $\left(R_{d}\right)$ is equal to $287.1 \mathbf{J}-\mathrm{kg}^{\mathbf{- 1}}-\mathrm{K}^{\mathbf{- 1}}$.
5. Levels of $\mathrm{CO}_{2}$ in the atmosphere have been increasing since the industrial revolution. Is the specific gas constant for dry air larger or smaller now than it was in 1800 ?
6. Explain why moist air is lighter than dry air (at the same pressure and temperature).
7. Assuming a scale height of 8.1 km , how many molecules are in $\mathbf{c m}^{3}$ of air at the altitude where the air pressure is 500 mb ? (Hint: density and the number of molecules per volume both decrease with height in the same way that pressure does)
8. How many oxygen molecules are there in a $\mathrm{cm}^{3}$ of air at the altitude where the air pressure is 500 mb ? Explain why airplane cabins are pressurized.
9. If the temperature at the ground is $15^{\circ} \mathrm{C}$ and the lapse rate is $4^{\circ} \mathrm{C} / \mathrm{km}$, what is the temperature at an altitude of 5000 m ?
10. If the atmosphere was incompressible (density constant at all altitudes), $100 \mathbf{k m}$ thick, and had a surface pressure of 1000 mb , at what altitude would the pressure be 250 mb ? Sketch the graph of pressure vs. altitude for this case and discuss how it compares with the real atmosphere.
11. Using $T(z)=T_{0}-\gamma z$ for the temperature in equation (12), show that this leads to equation (18).

[^0]:    ${ }^{1}$ Technically the derivative in (11) should be written as a partial derivative, $\partial p / \partial z$. For large-scale meteorological applications we can use a full derivative without losing accuracy. In this course we will use the full derivative form, since multivariable calculus is not required for this course.

