ESCI 340 - Cloud Physics and Precipitation Processes
Lesson 9 - Precipitation
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References:

A Short Course in Cloud Physics, 3rd ed., Rogers and Yau, Ch. 10
Microphysics of Clouds and Precipitation (2nd ed.), Pruppacher and Klett

Rainfall Rate

- Imagine a volume of air shown in Fig. 1

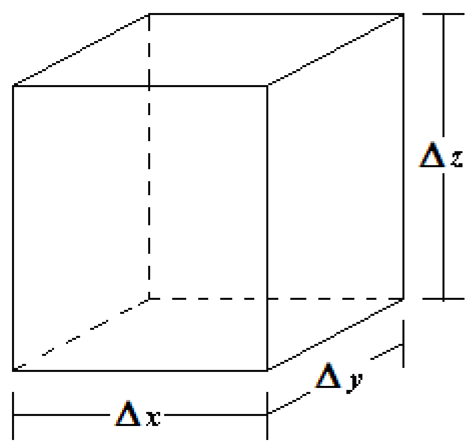

Figure 1: Volume of air, $V = \Delta x \Delta y \Delta z$.

- The mass of liquid droplets within the volume in the diameter range D to dD is dM, where M is the liquid water content.
- The time it takes for all of these droplets to fall through the bottom of the volume is

$$\Delta t = \frac{\Delta z}{u(D)}, \tag{1}$$

where $u(D)$ is the fall speed of the droplets having diameter D.

- The flux of water mass due to these droplets passing through the bottom of the volume is the mass contained in these droplets divided by the area divided by time. Therefore, the flux of water mass contained in droplets having diameters between D and dD is

$$dF = \frac{mass}{area \cdot time} = \frac{dM \Delta x \Delta y \Delta z}{\Delta x \Delta y \Delta t} = \frac{dM \Delta z}{\Delta t},$$

and from (1) this becomes

$$dF = u(D)dM. \tag{2}$$

- dM is related to the drop size distribution function via

$$dM = \frac{\pi}{6}\rho_l D^3 n_d(D)dD, \tag{3}$$

and so the mass flux of raindrops in the size range D to dD is

$$dF = \frac{\pi}{6}\rho_l u(D) D^3 n_d(D)dD. \tag{4}$$

- The total mass flux of the droplets is found by integrating (4) over all diameters, and is

$$F = \frac{\pi}{6}\rho_l \int_0^\infty u(D) D^3 n_d(D)dD. \tag{5}$$

- To convert the flux into a rate of accumulation of depth we use dimensional analysis as follows:

$$F = \frac{mass}{area \cdot time} = \frac{density \cdot volume}{area \cdot time} = \frac{density \cdot area \cdot depth}{area \cdot time} = \frac{density \cdot depth}{time}.$$

  - ***Rainfall rate*** is defined as *depth/time*. Therefore, we have

$$F = \rho_l R. \tag{6}$$

- Combining (5) and (6) we get the expression for rainfall rate in terms of the drop-size distribution,

$$R = \frac{\pi}{6}\int_0^\infty u(D) D^3 n_d(D)dD. \tag{7}$$

  - For a discrete droplet population the rainfall rate is

$$R = \frac{\pi}{6}\sum_{k=1}^{K} u_k D_k^3 N_k. \tag{8}$$

- We can bring the fall speed out from the integral in (7) by using the ***mean-value theorem***, so that

$$R = \frac{\pi}{6}\overline{u}\int_0^\infty D^3 n_d(D)dD, \tag{9}$$

where $\overline{u}$ is the ***volume-weighted mean terminal velocity***,[1]

$$\overline{u} = \frac{\int_0^\infty u(D) D^3 n_d(D)dD}{\int_0^\infty D^3 n_d(D)dD}. \tag{10}$$

[1] A volume-weighted mean is the same as a mass-weighted mean, since volume and mass only differ by ρ_l, which is a constant.

- For a discrete droplet population we could write (8) as

$$R = \frac{\pi}{6}\overline{u}\sum_{k=1}^{K} D_k^3 N_k,$$

where

$$\overline{u} = \frac{\sum\limits_{k=1}^{K} u_k D_k^3 N_k}{\sum\limits_{k=1}^{K} D_k^3 N_k}.$$

Marshall-Palmer Drop-size Distribution

- Raindrop size distributions can vary widely, but in general they often follow a distribution given by

$$n_d(D) = n_0 \exp(-\Lambda D). \tag{11}$$

  - Equation (11) is a form of the gamma distribution, $n_d(D) = aD^m \exp(-bD)$, introduced way back in Lesson 1 for cloud droplets.
  - For (11) we have $m = 0$, $a = n_0$, and $b = \Lambda$.

- Equation (11) is known as the ***Marshall-Palmer*** dropsize distribution.

- Observational studies show that n_0 has a nearly-constant value of approximately 0.08 cm^{-4}

- The value of the parameter Λ depends on the rainfall rate.

- Figure 2 shows the Marshall-Palmer distribution function for $n_0 = 0.08\ \text{cm}^{-4}$ and $\Lambda = 20\ \text{cm}^{-1}$.

- When plotted on a logarithmic y-axis the function appears as a line, with the slope given by $-\Lambda$, and the y-intercept given by n_0.

  - The parameter n_0 is therefore called the ***intercept parameter***, while Λ is called the ***slope factor***.

- Using the Marshall-Palmer distribution in our expression for rainfall rate, (9), results in

$$R = \frac{\pi n_0 \overline{u}}{\Lambda^4}. \tag{12}$$

- If we solve (13) for Λ we get

$$\Lambda = \left(\frac{\pi n_0 \overline{u}}{R}\right)^{1/4}, \tag{13}$$

  which allows us to estimate the slope parameter by measuring the rainfall rate.

- An empirical relation between rainfall rate an slope factor has been found to be

$$\Lambda = 41 R^{-0.21}, \tag{14}$$

  where R is in mm hr^{-1}, and Λ is in cm^{-1}.

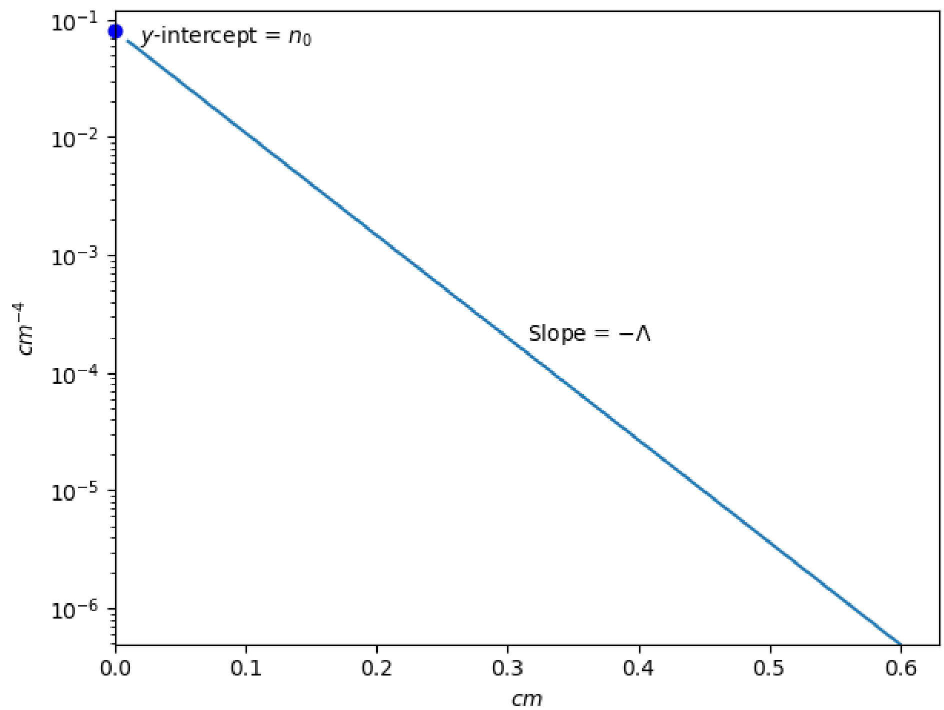

Figure 2: Plot of Marshall-Palmer distribution function for $n_0 = 0.08\ \text{cm}^{-4}$ and $\Lambda = 20\ \text{cm}^{-1}$.

Precipitation Types

Rain: Drops of water falling from a cloud, and having a diameter of greater than 0.5 mm

Drizzle: Liquid water drops having a diameter of less than 0.5 mm. You can often tell the difference between rain and drizzle because drizzle usually doesn't cause ripples in standing water puddles.

Snow: Ice crystals or aggregates of ice crystals. The shape (habit) of snowflakes varies with the amount of supersaturation at which they are formed.

Snow grains: Frozen equivalent of drizzle (from stratus clouds). Diameter less than 1 mm.

Sleet: Also called ***ice pellets***. Sleet is frozen raindrops.

Snow pellets: Also called ***graupel***. Larger than snow grain, but usually having diameter less than 5 mm. Snow pellets are crunchy and break apart when squeezed. Usually fall in showers from cumulus congestus clouds.

Hail: Hail begins as a snowflake that partially or completely melts, and then refreezes. But, instead of immediately falling to the ground, it gets caught in an updraft and can make several trips up and down through the cloud, each time accumulating more ice.

- Hail is only formed in very strong thunderstorms (cumulonimbus clouds).

- Hail has diameters greater than 5 mm. If smaller it is either small hail, snow pellets or ice pellets, depending on its appearance, hardness and crunchiness (see note below).
- NOTE: It is sometimes difficult to differentiate between hail, snow pellets, and sleet. Here are some rules to follow:
  - If it has a diameter larger than 5 mm it is usually hail.
  - If it has a diameter less than 5 mm, and is transparent, solid , and fairly spherical it is sleet. Sleet almost always falls from stratiform clouds, often in conjunction with a warm or stationary front.
  - If it has a diameter less than 5 mm, is opaque (not transparent), irregular, and crunches when squeezed, it is snow pellets.
    * Snow pellets generally fall from convective clouds such as cumulus congestus or cumulonimbus.
    * Snow pellets are easily crushable, and generally do not bounce.
  - If it has a diameter less than 5 mm and looks like hail, is hard, and bounces, then it is small hail. Small hail generally falls from cumulonimbus.

Glaze: Also called freezing rain, glaze forms when supercooled raindrops strike an object and instantly freeze on impact.

Rime: Forms in a manner similar to glaze, only it is caused by the freezing of supercooled cloud droplets rather than supercooled raindrops. It often forms feathery ice crystals on trees.

Exercises

1. Show that for a Marshall-Palmer drop-size distribution that the slope factor and the liquid water content are related via
$$\Lambda = \left(\frac{\pi\rho_l n_0}{M}\right)^{1/4}.$$
2. Show that using the Marshall-Palmer distribution in (9) results in
$$R = \frac{\pi n_0 \overline{u}}{\Lambda^4}.$$
3. In the formula for rainfall rate
$$R = \frac{\pi}{6}\overline{u}\int_0^\infty D^3 n_d(D)dD$$
where $\overline{u}$ is the volume-weighted mean terminal velocity. If instead you used the unweighted-mean terminal velocity
$$\overline{u} = \frac{1}{N}\int_0^\infty u(D)n_d(D)dD,$$

would the calculated rainfall rate be to low, or too high? Explain.