ESCI 340 - Cloud Physics and Precipitation Processes
Lesson 13 - Atmospheric Optical Phenomena
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References:

One of the best sources for information about atmospheric optics is the Atmospheric Optics website, *http://www.atoptics.co.uk*

Snel's Law and Refraction

- The index of refraction for a medium is defined as

$$m = c/\tilde{c}, \tag{1}$$

where c is the speed of light in a vacuum, and $\tilde{c}$ is the speed of light in the medium.[1]

- As light passes from one medium into another, there is both reflection and refraction.
- Refraction occurs because the wave fronts bend as they cross from one medium into another, causing a ray of light to bend. The ray bends toward the medium that has the slower speed of light (highest index of refraction).
- The bending of the ray is quantified by Snel's Law, which is stated mathematically as

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{m_2}{m_1}, \tag{2}$$

where θ_1 is the angle of incidence (and reflection), θ_2 is the angle of refraction, and m_1 and m_2 are the indices of refraction in the two mediums (see Fig. 1).

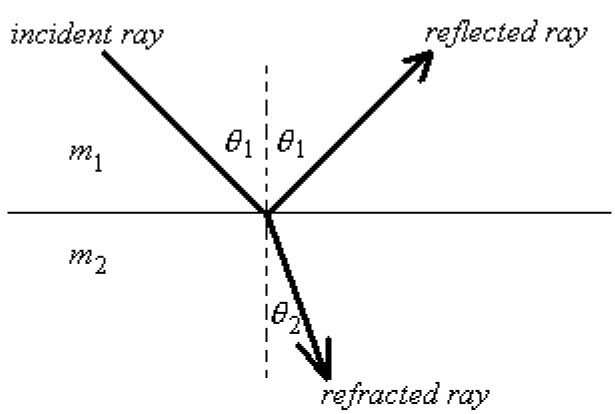

Figure 1: Illustration of Snel's Law.

[1] Technically the index of refraction is a complex number, with the real part defined by (1), and the imaginary part linked to the extinction or absorption of light in the medium.

- The amount by which a ray of light is deflected due to refraction can be quantified in one of two ways.
  - The ***bending angle***, θ', is the interior angle between the initial and final rays.
  - The ***deviation angle***, θ'', is the complement of the bending angle, $\theta'' = 180° - \theta'$.
  - The relationship between bending angle and deviation angle is illustrated in Fig. 2.

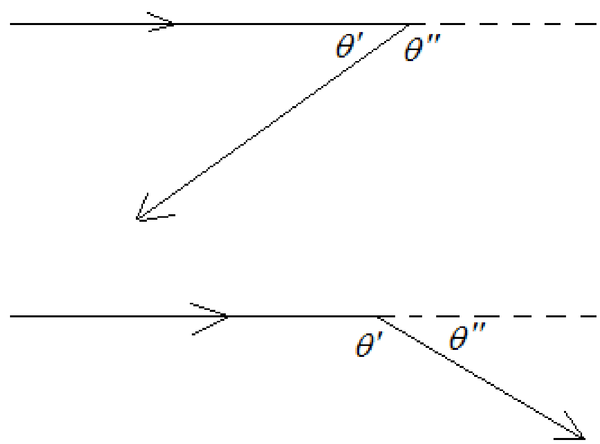

Figure 2: Relationship between bending angle, θ', and deviation angle, θ''.

Index of Refraction for Air

- Light travels faster through warm air than it does through cold air. Therefore, the index of refraction for visible light in air decreases with increasing temperature.
- Figure 3 shows $m - 1$ as a function of temperature for yellow light.

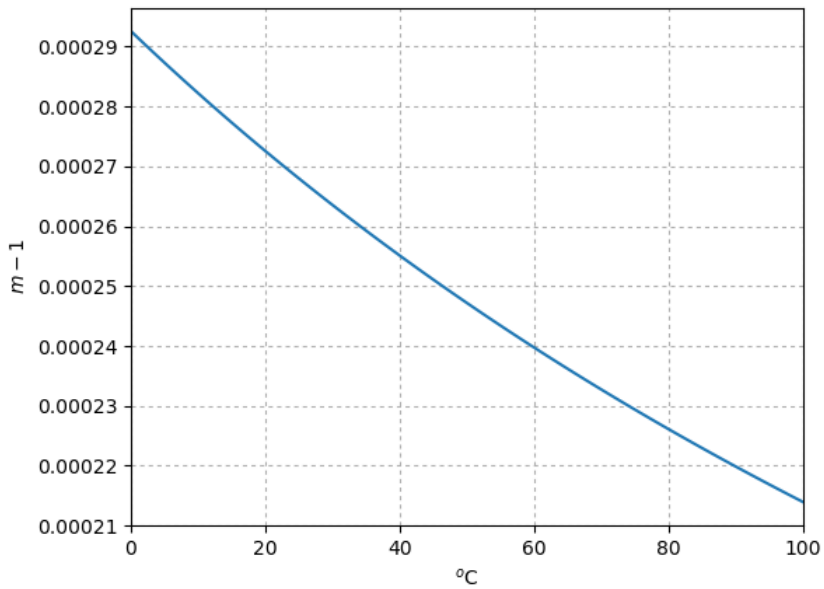

Figure 3: Index of refraction in air for yellow light ($\lambda = 0.58\mu$m) as a function of temperature at $p = 1013.25$ mb.

- The index of refraction for visible light in air decreases with increasing wavelength.
- Figure 4 below shows $m - 1$ as a function of wavelength.
- The change of the index of refraction in air over the visible wavelengths is very small, but shorter wavelengths do travel more slowly than do longer wavelengths, and will therefore bend more when passing from one medium to another.

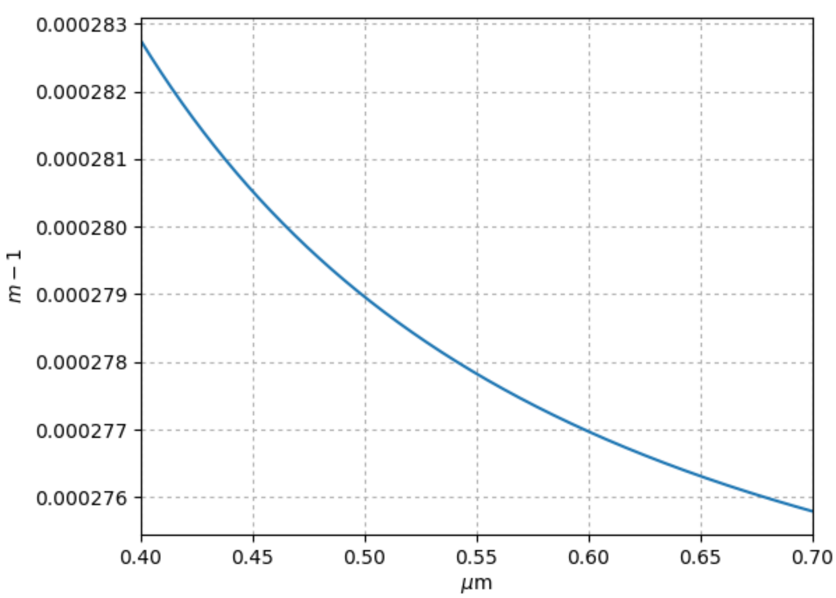

Figure 4: Index of refraction in air for visible light as a function of wavelength ($T = 15°$C; $p = 1013.25$ mb).

Mirages

- Mirages are caused by the bending of light rays either upward or downward near the ground.
- If the air near the ground is very hot, the light rays will bend upward.
  - This causes objects far away to appear inverted, and underneath their actual position
  - This results in an inferior image.
  - This is what causes hot asphalt to appear wet up ahead, because the sky appears as an inferior image in the road.
- If the air near the ground is very cold, the light rays bend downward.
  - This causes far away objects to appear above their actual position
  - This results in a superior image
  - This can actually allow objects that are over the horizon to be seen.

Primary Rainbows

- Rainbows are caused by the refraction and reflection of light by rain and cloud drops.
- The index of refraction for water is much larger than that for air, and also depends on wavelength.
- The index of refraction is greater for shorter wavelengths, so therefore, shorter wavelengths bend more.
- Figure 5 shows the path of a light ray through a spherical water droplet for a primary rainbow.

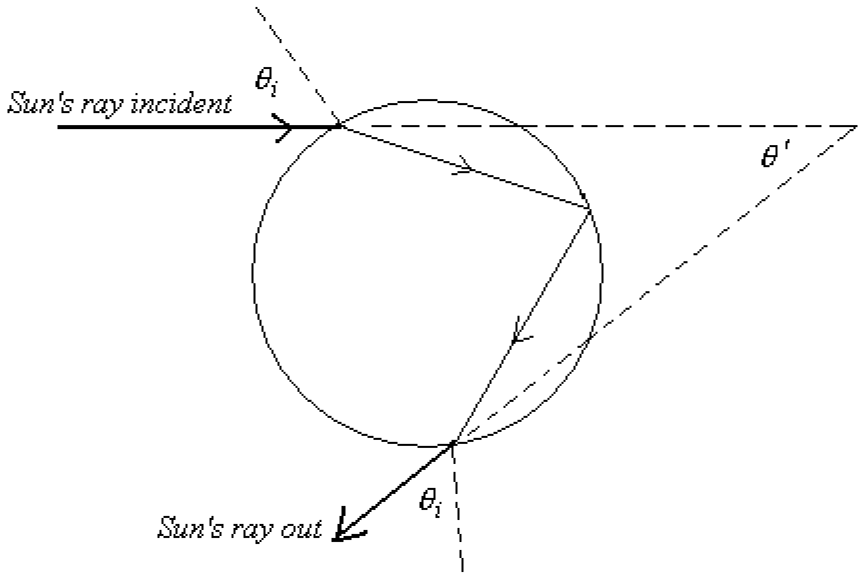

Figure 5: Path of light through a droplet for a primary rainbow.

- The interior angle θ' between the incoming and outgoing rays is called the *bending angle*.

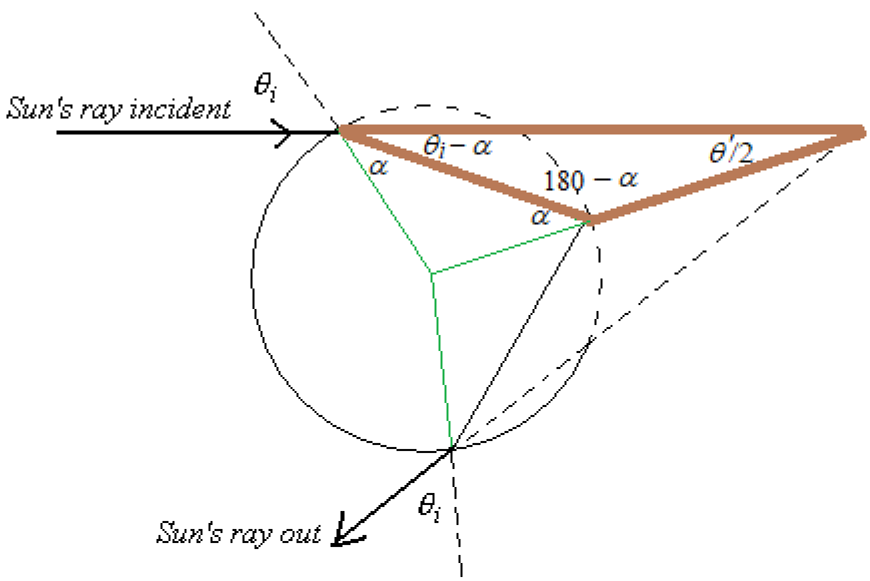

Figure 6: Diagram used for calculating (4).

- Figure 6 is an enhanced view of the path of light through a droplet.

– From the figure and using Snel's Law we have

$$\frac{\sin\theta_i}{\sin\alpha} = \frac{m_w}{m_a}, \tag{3}$$

where m_w is the refractive index of liquid water, and m_a is the refractive index of air.

– The angles of the bold, brown triangle must sum to 180°, and so

$$(\theta_i - \alpha) + (180^\circ - (\alpha + \theta'/2) = 180^\circ,$$

or

$$\theta' = 4\alpha - 2\theta_i. \tag{4}$$

– From (3) we know that

$$\alpha = \arcsin\left(\frac{m_a}{m_w}\sin\theta_i\right),$$

and so (4) becomes

$$\theta' = 4\arcsin\left(\frac{m_a}{m_w}\sin\theta_i\right) - 2\theta_i. \tag{5}$$

- There are an infinite number of possible incidence angles, and therefore paths, through the droplet. Figure 7 shows the bending angle as a function of incidence angle, calculated using (5).

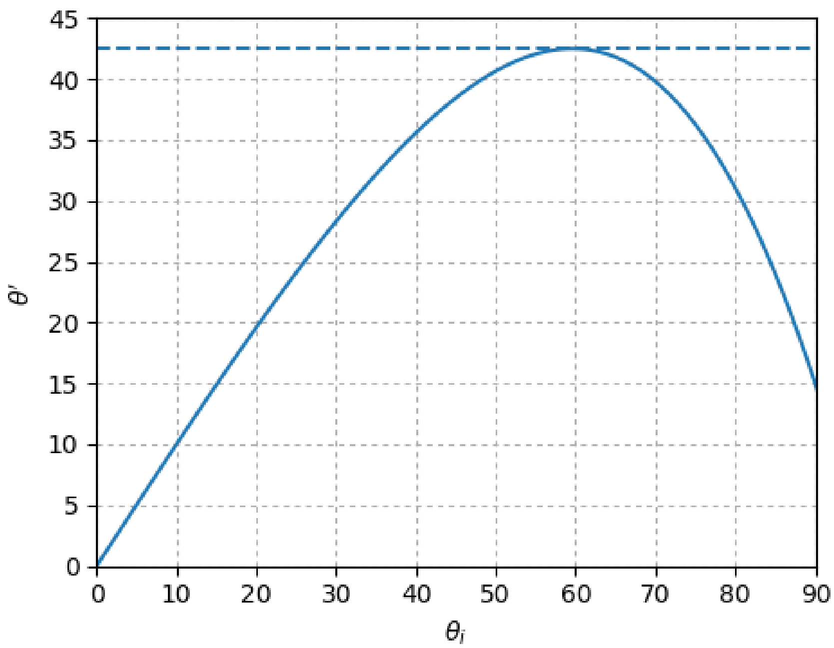

Figure 7: Primary rainbow bending angle as a function of incidence angle.

– Note from Fig. 7 that there is a certain incidence angle, $\tilde{\theta}_i$, at which the bending angle has a maximum value.

- At this particular incidence angle slight changes in θ_i lead to no change in θ'.
- At this special incidence angle, the light beam will be very concentrated. At other incidence angles, the light beam will not be as concentrated.
- It is only for this special incidence angle that a primary rainbow will be visible.
- $\tilde{\theta}_i$ is found by taking $\partial\theta'/\partial\theta_i = 0$ and solving for θ_i. The result is

$$\cos^2 \tilde{\theta}_i = \frac{1}{3}\left(\frac{m_w^2}{m_a^2} - 1\right). \tag{6}$$

- If we use the value $\tilde{\theta}_i$ in (5), then we obtain the value of the bending angle for our rainbow, which we will call $\tilde{\theta}'$.

- Since m_w is around 1.33 while m_a is around 1.0003, we can without much error, use

$$\frac{m_w}{m_a} \cong m_w.$$

Therefore, the equations for the incidence angle and bending angle for the primary rainbow (from (5) and (6)) are often written as

$$\cos^2 \tilde{\theta}_i = \frac{1}{3}(m_w^2 - 1) \tag{7}$$

and

$$\tilde{\theta}' = 4 \arcsin\left(\frac{1}{m_w} \sin \tilde{\theta}_i\right) - 2\tilde{\theta}_i. \tag{8}$$

- Table 1 shows the index of refraction for water for four different colors of light, and the associated values for the incidence and bending angles for a primary rainbow. The bending angle of red light is about 42.5°, while that for violet is about 40.7°.

	Violet	Green	Orange	Red
λ (μm)	0.4047	0.5016	0.5893	0.7061
m_w	1.3427	1.3364	1.3330	1.3300
$\tilde{\theta}_i$	58.9°	59.2°	59.4°	59.6°
$\tilde{\theta}'$	40.7°	41.6°	42.1°	42.5°

Table 1: Index of refraction, incidence angle, and bending angle for various colors of a primary rainbow.

- Because red light has a larger bending angle than does violet light, an observer perceives that a primary rainbow has red on the outside, and violet on the inside (see Fig. 8).

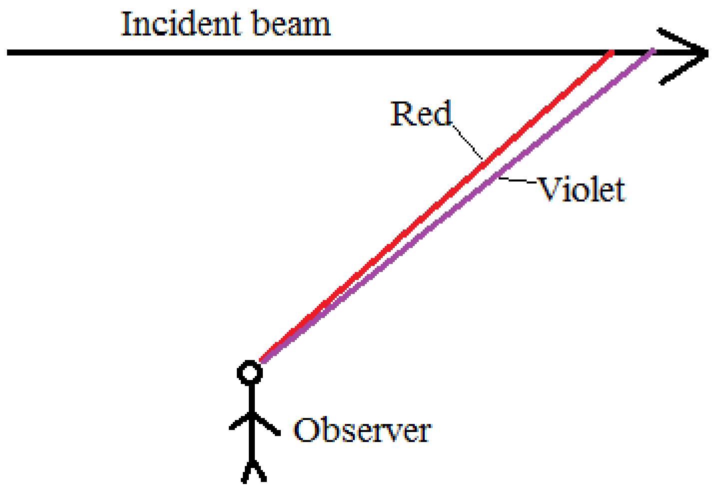

Figure 8: Viewing geometry for a primary rainbow. Red appears on top, because it has a larger bending angle.

  - Because each color bends through a slightly different angle, each color range you observe from a rainbow is from a different set or population of droplets. The violets you observe could not have been refracted through the same droplets as the reds you observe.

- If the Sun is more than 42.5° above the horizon then a primary rainbow cannot be seen by a ground-based observer.

Secondary Rainbows

- It is also possible to have a path through the water droplet that has two internal reflections, such as that shown in Fig. 9.

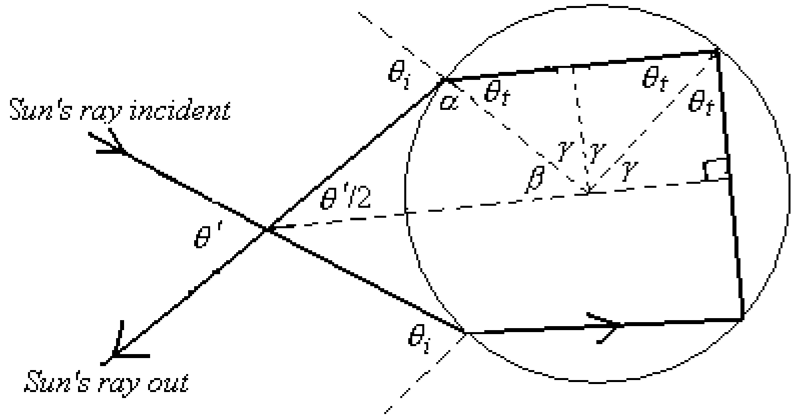

Figure 9: Path of light through a secondary rainbow.

- The relation between the bending angle, θ', and the incidence angle, θ_i, is worked out by deducing the following four independent relationships from Fig. 9,

$$\theta_i/2 + \alpha + \beta = 180^\circ \tag{9}$$
$$\theta_t + \gamma = 90^\circ \tag{10}$$
$$3\gamma + \beta = 180^\circ \tag{11}$$
$$\theta_i + \alpha = 180^\circ, \tag{12}$$

and eliminating the angles α, β, and γ from (9)–(12) to get

$$\theta' = 180^\circ + 2\theta_i - 6\theta_t. \tag{13}$$

  - From Snel's Law we have

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{m_w}{m_a} \cong m_w, \tag{14}$$

or

$$\theta_t = \arcsin\left(\frac{1}{m_w}\sin\theta_i\right). \tag{15}$$

  - Substituting (15) into (13) yields

$$\theta' = 180^\circ + 2\theta_i - 6\arcsin\left(\frac{1}{m_w}\sin\theta_i\right). \tag{16}$$

- As before, there are an infinite number of possible incidence angles; however, there is one special path for which $\partial\theta'/\partial\theta_i = 0$, and at this incidence angle the light will be concentrated. This special incidence angle is given by

$$\cos^2\tilde{\theta}_i = \frac{1}{8}(m_w^2 - 1), \tag{17}$$

and the bending angle for a secondary rainbow is therefore

$$\tilde{\theta}' = 180^\circ + 2\tilde{\theta}_i - 6\arcsin\left(\frac{1}{m_w}\sin\tilde{\theta}_i\right). \tag{18}$$

- Table 2 shows the incidence and bending angles for four colors for a secondary rainbow.

	Violet	Green	Orange	Red
λ (μm)	0.4047	0.5016	0.5893	0.7061
m_w	1.3427	1.3364	1.3330	1.3300
$\tilde{\theta}_i$	71.5°	71.7°	71.8°	71.9°
$\tilde{\theta}'$	53.4°	51.8°	50.9°	50.1°

Table 2: Index of refraction, incidence angle, and bending angle for various colors of a secondary rainbow.

- For a secondary rainbow the shorter wavelengths have a larger bending angle. Therefore, the order of the colors of for a secondary rainbow is reversed from that of the primary rainbow.
- Secondary rainbows are also much dimmer than primary rainbows, because every time the light refracts and/or reflects, some of the radiance is lost out of the beam.
- If the Sun is more than 50° above the horizon then a secondary rainbow is not visible to a ground-based observer.

Alexander's Dark Band

- The maximum bending angle for light experiencing only one reflection inside a droplet is about 42°.
- The minimum bending angle for light experiencing two reflections inside a droplet is about 50°.
- Between these two limits there is no light being bent from the droplets toward the observer, as shown in Fig. 10. The result is a region of relative darkness between the primary and secondary rainbows.
- The band of darkness is known as *Alexander's dark band*, after Alexander of Aphrodisias, who wrote about this phenomenon in the year 200.

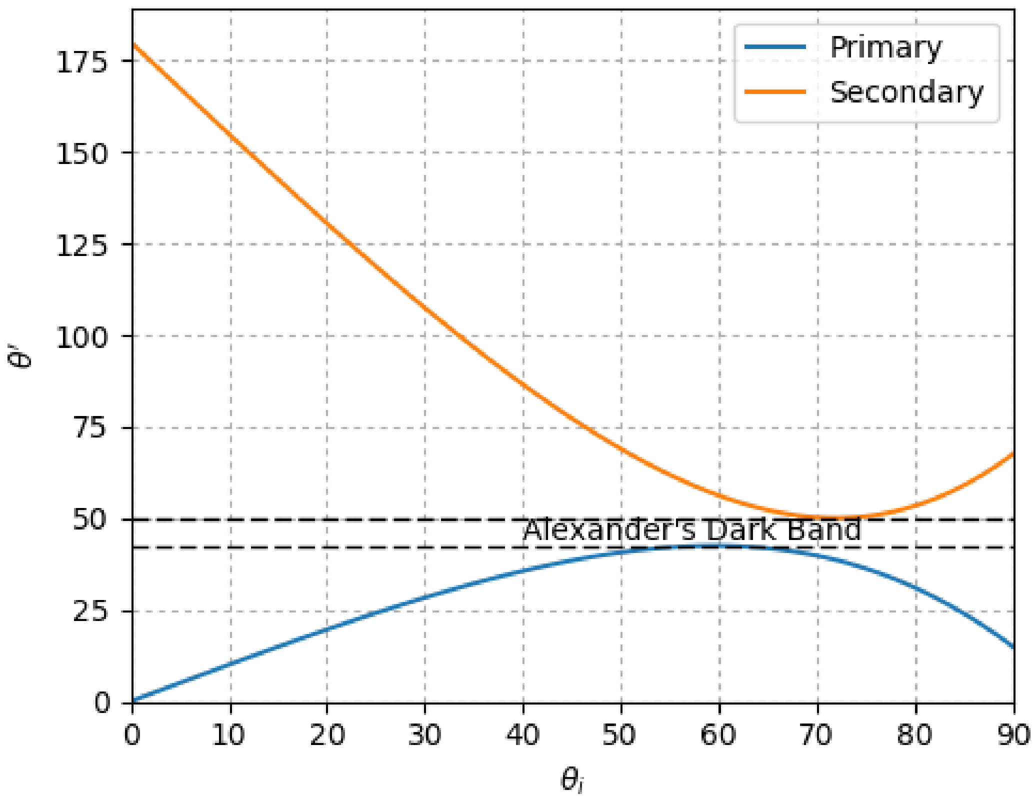

Figure 10: Bending versus incidence angle for primary and secondary rainbows. There are no bending angles between 42° and 50° for either type of rainbow, resulting in relative darkness between the bows.

Other Types of Rainbows

- Higher order rainbow involve more than two internal reflections inside the water droplet.
  - Higher order bows are much fainter, because every internal reflection results in a loss of light from the beam.
  - Many higher order bows also have bending angles of more than 90°, meaning you have to be looking toward the sun to see them, which makes them more difficult to see.
- Supernumerary rainbows are concentric arcs that appear on the inside of a primary rainbow.
  - Supernumeraries appear very close to each other, and are often greenish or purplish.
  - If two rays of the same color take very slightly different paths through the droplet, then they may either constructively interfere with each other, enhancing the

brightness, or they may destructively interfere with each other, decreasing the brightness of the beam.

  - To form supernumeraries the droplets must be fairly uniform, nearly spherical raindrops (so therefore, small droplets).

- Sometimes the top of a rainbow will appear to be 'twinned'. This is thought to be due to the nonspherical shape of large raindrops as they fall, causing the light rays to bend through a different angle than they would through a spherical drop.

- See *http://www.atoptics.co.uk* for more detailed information on higher order, supernumerary, and other types of rainbows.

Corona

- Corona appear as colored rings around the sun or moon.

- They are smaller than the 22° halo.

- Corona are formed by *diffraction* of light through cloud droplets, and sometimes through cloud ice crystals.

Ice Crystals as Prisms

- Ice crystals come in many different shapes, or habits (plates, columns, needles, etc.).

  - The habit depends on the temperature and humidity at which the crystal forms.
  - Most ice crystals are six-sided (hexagonal).

- Ice crystals act as prisms, refracting the light as it passes through the crystal.

- Figure 11 shows some possible symmetric paths of light through a hexagonal column.

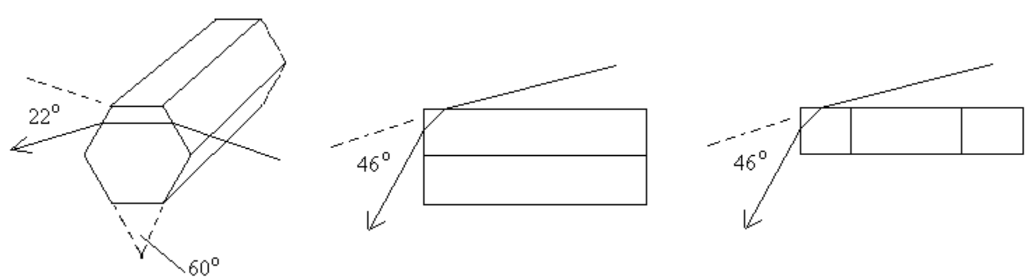

Figure 11: Paths of light through a hexagonal ice crystal column or plate.

- One path is for the light to pass laterally across the crystal, shown on the left of Fig. 11.

  - In this case, the crystal acts like a prism with an angle of 60°.

- The other possible path is for the light to pass longitudinally along the crystal, as shown on the right of Fig. 11.
  - In this case, the crystal acts like a prism with an angle of 90°.
- A third path, shown in Fig. 12, is impossible for ice crystals because the index of refraction for ice is too large.

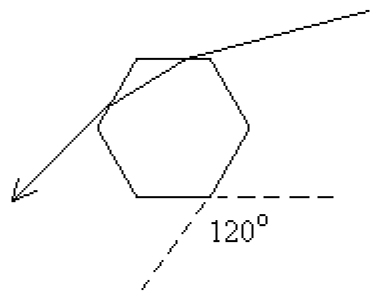

Figure 12: Invalid path of light through a hexagonal ice crystal column.

- The deviation angle θ'' from a prism depends on the incidence angle θ_i, the index of refraction of the prism, m, and the angle of the prism, A. The formula is

$$\theta'' = \theta_i - A + \arcsin\left(m \sin\left[A - \arcsin\left(\frac{\sin\theta_i}{m}\right)\right]\right), \tag{19}$$

  the derivation of which is shown in the Appendix.
- Figure 13 shows the relationship between deviation angle and incidence angle from (19) for the two possible paths through an ice crystal for three different wavelengths of light; 0.70μm, 0.58μm, and 0.40μm. The indices of refraction for these wavelengths are 1.3070, 1.3100, and 1.3195.[2]

[2]Data are from Warren, S. G., 1984: Optical constants of ice from theultraviolet to the microwave. *Appl. Opt.*, **23**, 1206-1224.

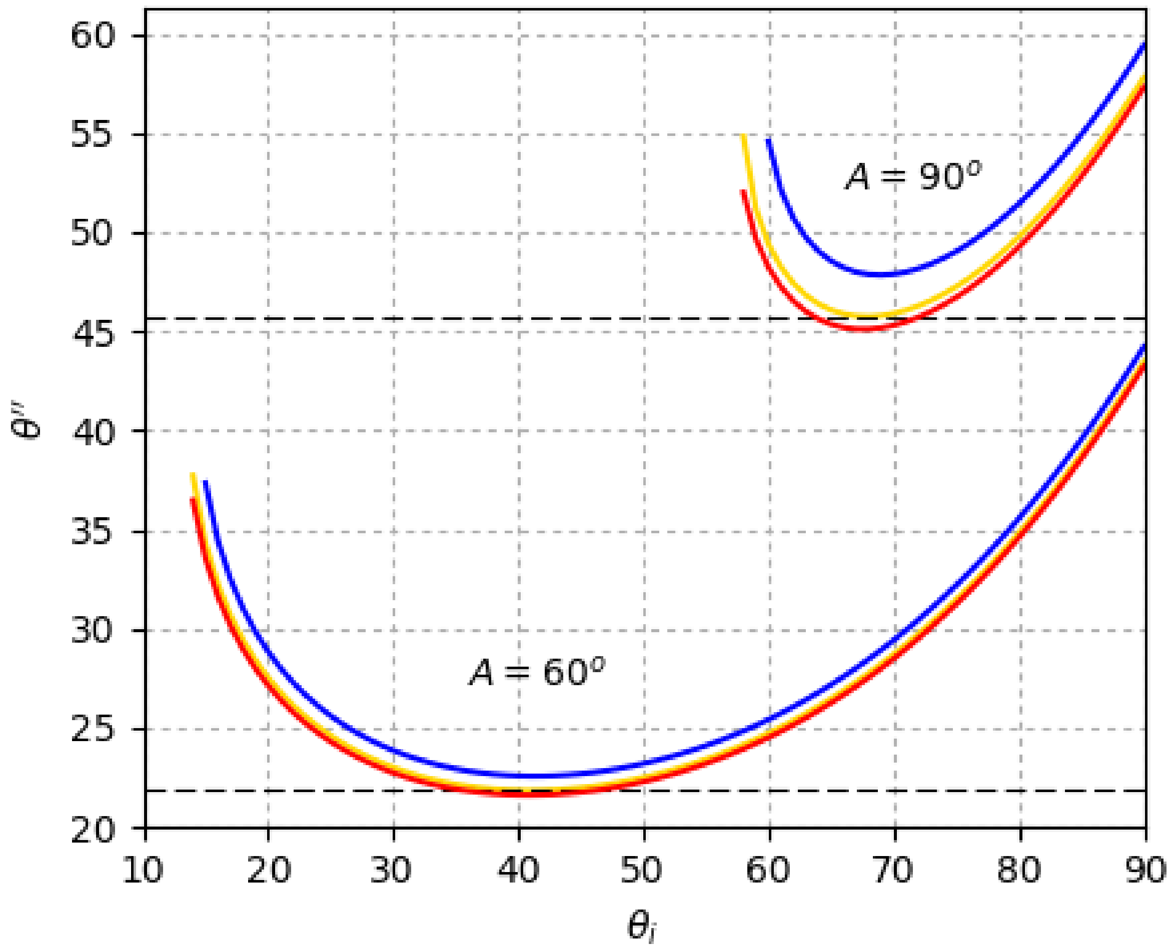

Figure 13: Deviation angle versus incidence angle for ice crystals. The three colored lines are for wavelengths corresponding to red, yellow, and violet light (0.70μm, 0.58μm, and 0.40μm). The minimum deviation angle for yellow light (shown by the dashed horizontal lines) is either 22° or 46°, depending on which path through the crystal the ray traverses.

- Note that there is a minimum deviation angle at which $\partial\theta''/\partial\theta_i = 0$, and for which the beam through the prism will be concentrated. This concentrated path also happens to be the path that passes symmetrically through the prism (also derived in the Appendix).

- The minimum deviation angle is given by

$$\theta''_{min} = 2\arcsin\left[m\sin\left(\frac{A}{2}\right)\right] - A. \tag{20}$$

- The index of refraction for ice is wavelength-dependent, and is larger for shorter wavelengths.

– Because of this wavelength dependence, red light has the smallest deviation angle, while violet light has the largest deviation angle.

– When sunlight refracts through ice crystals, the red light appears on the side of the phenomenon that is closer to the sun, while the blue/violet light appears on the side further from the sun.

Ice Crystal Optical Phenomena

- Halos, sun dogs (also called parhelia), sun pillars, and various arcs are caused by the reflection or refraction of sunlight from ice crystals.
- Figure 14 illustrates the locations with respect to the sun of some of these phenomena.

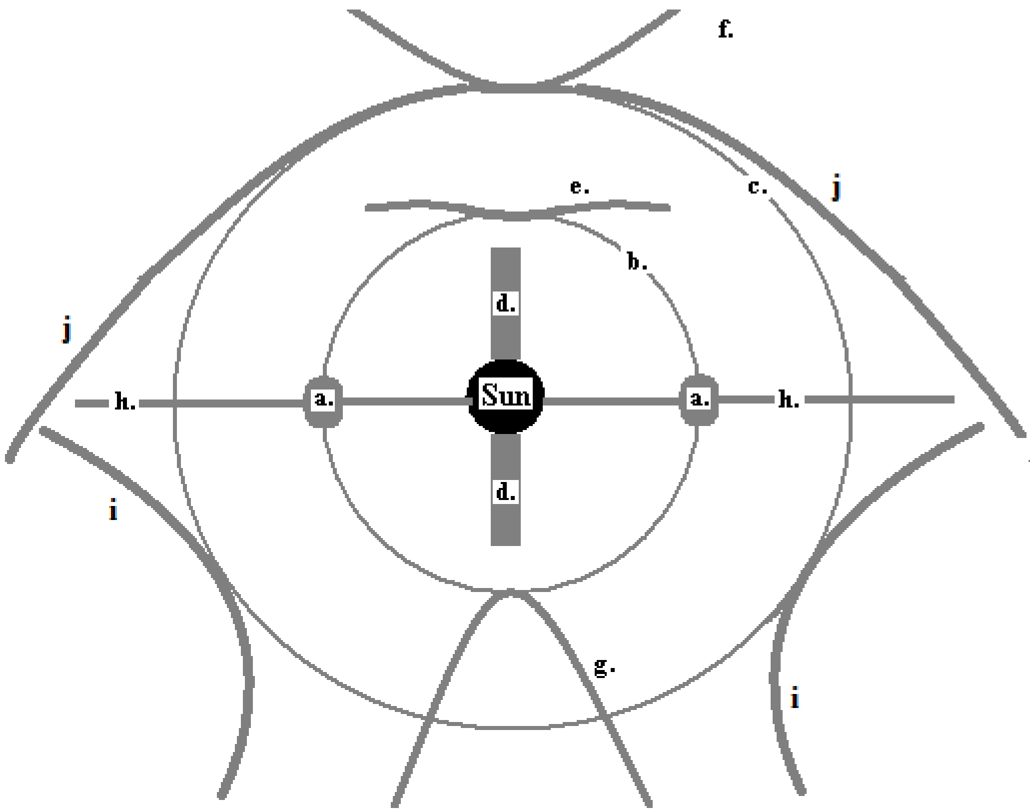

Figure 14: Schematic showing various optical phenomena associated with ice crystals: a. parhelia (sun dogs); b. 22° halo; c. 46° halo; d. sun pillars; e. upper-tangent arc; f. circumzenithal arc; g. lower-tangent arc; h. parhelic circle; i. infraleral arc; j. supralateral arc.

- Which phenomena occur depends on the shape and orientation of the ice crystals.

Halos: Halos occur when columnar crystals are randomly oriented. This results in a ring around the sun or moon.

- Halos are displaced either 22° or 46°, depending on the path of light through the crystal.
- The 46° halo is actually quite rare. Many reports of partial 46° halos are instead pieces of supralateral or infralateral arcs, which also occur near 46° from the sun.

Sun dogs (*parhelia*): Sun dogs appear 22° to either side of the sun, and occur when the ice crystals are hexagonal plates, rather than columns. The plates must be fairly uniformly oriented horizontally.

Tangent arcs: Tangent arcs touch the 22° degree halo at either that top (*upper tangent arc*) or bottom (*lower tangent arc*) of the halo.

- Tangent arcs form when hexagonal columns are nearly uniformly oriented with the long axis parallel to the horizon (the rotation of the long axis about the vertical is unimportant, as long as the long axis is nearly horizontal).
- The rays through the ice crystals are most concentrated at the minimum deviation angle of 22°, but the larger deviations also occur, which gives rise to the 'wings' of the tangent arcs.
- The shape of the tangent arc's wings change depending on how far the sun is above the horizon.
- If the sun angle is less than 29° the upper and lower tangent arcs merge to form the ***circumscribed halo***.

Circumzenithal arc: The circumzenithal arc is formed by light rays bending through hexagonal plate crystals that are fairly uniformly oriented (the same conditions favorable for sun dog formation).

- For the circumzenithal arc the light path through the crystals is through the top plate face and then out one of the sides (a prism with $A = 90°$) as shown in Fig. 15.
- Refering to Fig. 13 we see that for the $A = 90°$ prism path through ice, that the minimum angle of incidence, θ_i is about 58°. The relation between sun angle and incidence angle is $\psi = 90 - \theta_i$. Therefore, the sun angle must be less than 32° in order for a circumzenithal arc to form.
- Figure 13 also shows us that the minimum deviation, and hence the most concentrated beam, occurs when the incidence angle is about 68°, which equates to a sun angle of 22°. Thus, circumzenithal arcs are most vivid and pronounced when the sun is 22° above the horizon.

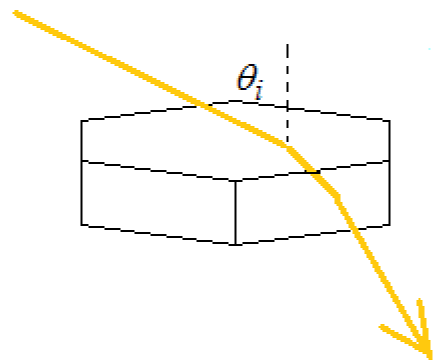

Figure 15: Path of light through a horizontally-oriented hexagonal plate, resulting in formation of a circumzenthal arc.

Parhelic circle: The parhelic circle forms from external and internal reflections from the vertical faces of crystals. It is always at the same height above the horizon as the sun itself.

Supralateral and infralateral arcs: These arcs are formed by light passing through horizontally-oriented hexagonal columns, as shown if Fig. 16.

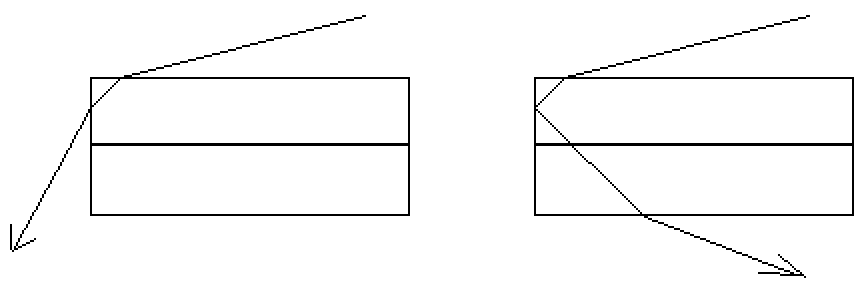

Figure 16: Path of light through a horizontally-oriented hexagonal hexagonal column, resulting in formation of supralateral and infralateral arcs.

- There may be internal reflections along the path.
- Supraleral arcs cannot form at sun angles greater than 32°, for the same reason that circumzenithal arcs cannot form at higher sun angles.
- Supralateral and infralateral are rare, but are more common than the 46° halo. Most reports of pieces of the 46° halo are likely supralateral or infralateral arcs.

Sun pillars: Sun pillars are formed by reflections off of the upper or lower faces of plate-like crystals.

Appendix - Deviation Angle for a Prism

- Figure 17 shows a ray passing through a prism of angle A at an arbitrary incidence angle θ_i. The ray exits at an angle θ_o.

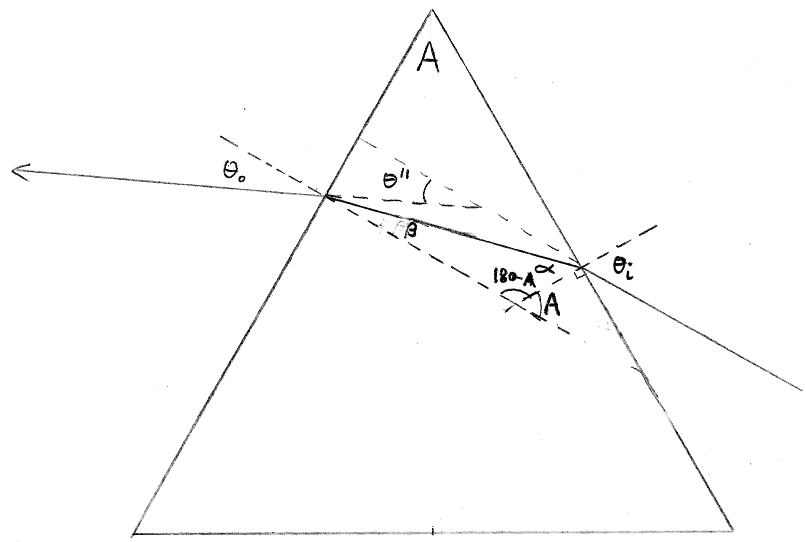

Figure 17: Ray passing through a prism at an arbitrary incidence angle.

- From the diagram we can deduce the following relations:

$$A = \alpha + \beta \tag{21}$$

$$\theta'' = (\theta_o - \beta) + (\theta_i - \alpha). \tag{22}$$

- From Snel's law we know that θ_o is given by

$$\theta_o = \arcsin(m \sin \beta), \tag{23}$$

so that (22) becomes

$$\theta'' = \arcsin(m \sin \beta) - \beta + \theta_i - \alpha. \tag{24}$$

- From (21) we can write $\beta = A - \alpha$, and using this in (24) we get

$$\theta'' = \arcsin[m \sin(A - \alpha)] - (A - \alpha) + \theta_i - \alpha,$$

which simplifies to

$$\theta'' = \arcsin[m \sin(A - \alpha)] - A + \theta_i. \tag{25}$$

- Finally, from Snel's law we have

$$\alpha = \arcsin\left(\frac{\sin \theta_i}{m}\right). \tag{26}$$

so that (25) becomes

$$\theta'' = \arcsin\left(m\sin\left[A - \arcsin\left(\frac{\sin\theta_i}{m}\right)\right]\right) - A + \theta_i. \tag{27}$$

- A plot of deviation angle versus incidence angle, Fig. 13, shows that there is only one particular incidence angle for which the deviation is minimized. To find this incidence angle we could differentiate (27) with respect to θ_i, set the result equal to zero, and then solve for θ_i; however, this would be very tedious.

- Instead, we can recognize that at the minimum deviation, θ_i must equal θ_o. For if it weren't, then if we reversed the ray in Fig. 17 the angle of incidence would be θ_o, and yet we would get the same deviation as for the forward ray. So, there would be two incidence angles giving us the same deviation! That does not square with our graph, and so we can deduce that at minimum deviation, $\theta_i = \theta_o$. Therefore, it is the symmetric path through the prism that gives the minimum deviation of the ray.

- For this symmetric path we have $\alpha = \beta = A/2$, and so from (22) we get

$$\theta''_{min} = 2\theta_i - A. \tag{28}$$

- From Snel's Law,

$$\sin\theta_i = m\sin\alpha = m\sin\left(\frac{A}{2}\right),$$

or

$$\theta_i = \arcsin\left[m\sin\left(\frac{A}{2}\right)\right]. \tag{29}$$

- Putting (29) into (28) gives

$$\theta''_{min} = 2\arcsin\left[m\sin\left(\frac{A}{2}\right)\right] - A.$$