Cloud Physics and Precipitation Processes
Lesson 1 - Cloud Types and Properties
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References:

Glossary of Meteorology, 2nd ed., American Meteorological Society
A Short Course in Cloud Physics, 3rd ed., Rogers and Yau, Ch. 5

Cloud Formation

- When air becomes saturated with water vapor, any excess water vapor condenses to form clouds.
- The air can become saturated either by:
  - addition of water vapor
  - cooling the air
- A common way for air to become saturated is for it to be lifted and adiabatically cooled via one of the four methods of lifting:
  - orographic lifting
  - frontal wedging
  - convergence
  - convective lifting
- In order to condense, there must be a surface for the water to condense onto. In the atmosphere, tiny dust, dirt, or smoke particles serve as these surfaces. They are known as condensation nuclei.
  - In the absence of condensation nuclei the relative humidity can get up to 400% without condensation occurring.
  - If the relative humidities over 100%, the air is said to be supersaturated.
- Not all particles in the atmosphere can be condensation nuclei. Only those that have an affinity for water (called hygroscopic nuclei) are effective as condensation nuclei.
- Clouds are composed of a large number of very small droplets of water. The droplets are so small that they do not fall, but remain suspended in the air.
- A typical cloud will have a droplet concentration of a few hundred per cubic centimeter, or about 500,000 droplets in a 2 liter soft-drink bottle sized parcel.

Cloud Classification

Clouds are classified according to *height* and *form*:

- Height of base above ground, h:

  Low: $h < 6000$ ft

  Middle: $6000 \text{ ft} \leq h \leq 20,000$ ft

  High: $h > 20,000$ ft

  Vertically developed: Bases are low, but cloud can grow to great heights.

- Form:

  Stratiform: Spread horizontally, little vertical growth

  Cumuliform: Billowy, with vertical growth

  Cirriform: Comprised of ice crystals

  A cloud may have more than two forms.

Cloud Descriptions

The descriptions below are taken either wholly or in part from the *Glossary of Meteorology*.

- High clouds:

  Cirrus: Delicate, icy filaments. Often form *uncinus*, or "mare's tails".

  Cirrostratus: Transparent veil, often smooth and covering much of sky. May produce *halo*, *parhelia*, or other optical effects.

  Cirrocumulus: Small white patches. May appear cellular or with ripples. Often has a regular pattern. Sometimes referred to as "mackerel sky" for fish scale appearance.

- Middle clouds:

  Altocumulus: Similar to cirrocumulus, but are lower, have larger cells, and are composed of water drops rather than ice crystals.

  Altostratus: Grayish smooth clouds covering most of the sky. Sun is usually visible, but not distinct, as though you are looking at it through frosted glass. No halos, parhelia, or other similar optical phenomena.

- Low clouds:

  Stratus: Low, uniform cloud that covers much of the sky. It may produce drizzle or snow grains.

Stratocumulus: Similar to stratus, though the bottom has long, parallel rolls or cellular structure.

Nimbostratus: Forms when stable air is forced to rise. A dark, low, uniform cloud, similar to stratus, but with continuous precipitation.

- Clouds of vertical development:

Cumulus Humilis: Individual, puffy masses that can grow vertically into towers or domes.

Cumulus Mediocris: Moderate vertical development. No precipitation.

Cumulus Congestus: Strongly sprouting cumulus with sharp outlines and sometime with great vertical development. Often referred to as *towering cumulus*. May produce heavy rain showers, especially in The Tropics.

Cumulonimbus: Cumulus clouds with great vertical development (usually fills the entire troposphere). Distinguished from cumulus congestus by presence of ice crystals (***glaciation***). Produces rain, and often, lightning. May also produce hail. An anvil head is sometimes formed at the top where the cloud presses against the level of neutral buoyancy (LNB).

Liquid Water Content

- Liquid water content is denoted as M, and is the mass of liquid per volume of air.
- In MKS units liquid water content is kg m^{-3}, but to avoid small numbers it is usually expressed as g m^{-3}.
- Liquid water content varies geographically, and also with the type of cloud, but some general characteristics are:

Cloud Type	M (g m^{-3})
Fog	0.05 – 0.5
Cumulus (early stage)	0.2 – 0.5
Cumulus (late stage)	0.5 – 1
Cumulus congestus/cumulonimbus	0.5 - 3
Altocumulus/altostratus	0.2 – 0.5
Stratus/stratocumulus	0.1 – 0.5
Nimbostratus	0.2 – 0.5

- Strong updrafts can produce and sustain large liquid water contents, with some observations of 5 – 14 g m^{-3}.
- ***Adiabatic liquid water content***, M_a, is the liquid water content that would be produced by moist-adiabatic cooling of a saturated air parcel. It assumes that the parcel starts at the base of the cloud at saturation, and that all of the condensation remains in the parcel, and doesn't fall out.

- Due to ***entrainment*** of drier air along the edges of the cloud the actual liquid water content in a cloud is usually significantly less than the adiabatic liquid water content.
  - Typical ratios of M/M_a are 0.1 – 0.6.
  - The ratio M/M_a can be close to unity near cloud base, but generally decreases with height.
  - The ratio M/M_a is larger for wider clouds, since the effects of entrainment are smaller in these clouds due to their lower ratio of surface area to volume.

Relative Humidity and Saturation Ratio

- Relative humidity is defined as the ratio of vapor pressure to saturation vapor pressure, expressed as a percentage.
- Relative humidity RH inside of clouds is usually between 98% and 102%.
- Near cloud edges, where entrainment is large, RH may be as low as 70%.
- Saturation ratio, S, is relative humidity expressed as a fraction rather than as a percent. So, a relative humidity of 85% is the same as a saturation ratio of 0.85.

Cloud Droplet Size Distribution

- The number of droplets per unit volume is called the number density, and is denoted by N.
  - The units of N are m^{-3}.
  - The units are really number per meter cubed, but we do not usually write out the word 'number'.
- Clouds contain droplets of different sizes. The total number density is found by summing the number densities of the different diameters,

$$N = \sum_{k=1}^{K} N_k, \tag{1}$$

  where $N_k = N(D_k)$ is the number density of droplets having diameter D_k, and K is the total number of separate droplet sizes.

- If there are many different drop sizes spaced closely together, the droplet spectrum can be represented by a continuous distribution function, $n_d(D)$, and the total number of droplets per unit volume is found by integrating the distribution function over all diameters,

$$N = \int_0^\infty n_d(D)dD. \tag{2}$$

- The function $n_d(D)$ is called the drop-size distribution function, and has units of m^{-4}.
- The differential of number density is

$$dN = n_d(D)dD. \tag{3}$$

- The differential dN is interpreted as the number density of droplets in the diameter range between D and $D + dD$.
- The number density of drops having diameters in the range between D_1 and D_2 is given by the integral

$$N_{D_1:D_2} = \int_{D_1}^{D_2} n_d(D)dD. \tag{4}$$

- The probability of randomly selecting a droplet that has a diameter between D_1 and D_2 is

$$P(D_1 : D_2) = \frac{N_{D_1:D_2}}{N} = \int_{D_1}^{D_2} \frac{n_d(D)}{N} dD = \int_{D_1}^{D_2} p_d(D)dD, \tag{5}$$

  so that $\frac{n_d}{N}$ can be though of as the prbability density function for the drop diameter,

$$p_d(D) = \frac{n_d}{N}. \tag{6}$$

- From probability theory we know that the mean or expected value of a variable x can be found from its probability density function, $p(x)$ via

$$\overline{x} = \int_0^{\infty} xp(x)dx. \tag{7}$$

  Using (6) and (7) we can find that the mean diameter of the droplets in a continuous distribution is given by

$$\overline{D} = \frac{1}{N} \int_0^{\infty} Dn_d(D)dD. \tag{8}$$

- For a discrete distribution the mean diameter would be[1]

$$\overline{D} = \frac{1}{N} \sum_{k=1}^{K} D_k N_k. \tag{9}$$

[1] Any formula containing an integral of the continuous distribution function can be converted into a sum over the discrete diameter bins.

Liquid Water Content from Droplet Size Distribution

- We can find the liquid water content from the drop size distribution as follows:
  - The mass of a droplet of diameter D is
  $$m = \frac{\pi}{6}\rho_L D^3, \tag{10}$$
  where ρ_L is the density of liquid water.
  - The number density of droplets having diameters between D and $D+dD$ is given by (3), so if we multiply (3) by (10) we have
  $$dM = mdN = \frac{\pi}{6}\rho_L D^3 n_d(D)dD. \tag{11}$$
  - Integrating (11) between diameters D_1 and D_2 gives the liquid water content contributed by droplets in this size range,
  $$M_{D_1:D_2} = \frac{\pi}{6}\rho_L \int_{D_1}^{D_2} D^3 n_d(D)dD. \tag{12}$$
- The total liquid water content for all droplets is found by integration (11) from 0 to ∞,
$$M = \frac{\pi}{6}\rho_L \int_0^{\infty} D^3 n_d(D)dD. \tag{13}$$
- For a discrete distribution the total liquid water content is
$$M = \frac{\pi}{6}\rho_l \sum_{k=1}^{K} D_k^3 N_k. \tag{14}$$

Actual Drop Size Distributions for Clouds

- The drop size distributions of actual clouds can be quite complex, and vary from cloud to cloud.
- Some distributions have more than one peak, while others have a single peak.
- In general, maritime cumulus clouds have larger drops and broader distributions than do continental clouds.
- In many instances the drop-size distribution is represented very closely by a form of the ***gamma distribution***, having the form
$$n_d(D) = aD^2 \exp(-bD) \tag{15}$$
where a and b are constants.

The Gamma Function

- A common integral that appears when working with drop size distributions is of the form $\int_0^\infty x^m \exp(-bx)dx$.
- The integral evaluates to

$$\int_0^\infty x^m \exp(-bx)dx = \frac{\Gamma(m+1)}{b^{m+1}}, \tag{16}$$

where $\Gamma(p)$ is the ***gamma function***, defined as

$$\Gamma(p) = \int_0^\infty t^{p-1}e^{-t}dt. \tag{17}$$

- The gamma function has the following recursion property,

$$\Gamma(p+1) = p\Gamma(p). \tag{18}$$

- If p is a positive integer then

$$\Gamma(p+1) = p!\ . \tag{19}$$

- This means that for positive integer values of m the integral (16) becomes

$$\int_0^\infty x^m \exp(-bx)dx = \frac{m!}{b^{m+1}} \qquad m = 0, 1, 2, 3, \ldots, \infty. \tag{20}$$

Distance Between Droplets

- The mean distance between droplets in a population is given by

$$\overline{r} = 0.554N^{1/3}. \tag{21}$$

- If interested in how this is derived, see *https://blogs.millersville.edu/adecaria/files/2021/11/Mean_Distance_Between_Cloud_Drops.pdf*.

Closing Words on Cloud Properties

- Parameters like drop size distribution, liquid water content, and distance between drops vary greatly between clouds, as well as within individual clouds. These parameters also vary with time as a cloud evolves and develops.
- Clouds have an important role in the radiation balance of the Earth, because they not only reflect, scatter, and absorb solar radiation, but they also absorb terrestrial radiation.

- Changes in cloud parameters affect the radiation balance. For example, clouds composed of numerous, small droplets are more reflective (have a higher albedo) than those comprised of fewer, larger droplets. Thus, two clouds may have the same liquid water content, but may have very different effects on solar radiation.
- Understanding cloud microphysical processes, and being able to represent these processes in models, is a key problem for both numerical weather prediction and climate modeling.

Exercises

1. Show that the surface area density of all drops having diameters between D_1 and D_2 is given by
$$A_{D_1:D_2} = \pi \int_{D_1}^{D_2} D^2 n_d(D) dD.$$
2. Assume a population of cloud droplets follows the gamma distribution, (15), with $a = 4.53 \times 10^{24}$ m^{-6} and $b = 2.35 \times 10^5$ m^{-1}.
   (a) What is the number density of the droplets (in cm^{-3})?
   (b) What is the liquid water content (in g m^{-3})?
   (c) What is the surface area density of the droplets (in cm^2 m^{-3})?
   (d) What is the mean drop diameter (in μm)?
   (e) What is the mean distance between droplets (in mm)?