ESCI 485 – Air/sea Interaction
Lesson 10 – Storm Surge
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Reference: ***Atmosphere-Ocean Dynamics*, Gill**

GENERAL

- **Storm surge refers to the raising of sea-level along a shore due to the transport of water toward the shore.**
- **Storm surge is not simply a case of the wind pushing water toward the shore. On a rotating planet, the water transport is directed at an angle to that of the wind (due to the Ekman transport).**

UNIFORM, ALONGSHORE WIND STRESS

- **Storm surge can be visualized and modeled using the shallow-water equations. These are**

$$\frac{Du}{Dt} = -g\frac{\partial \eta}{\partial x} + fv + \tau_x / \rho H$$
$$\frac{Dv}{Dt} = -g\frac{\partial \eta}{\partial y} - fu + \tau_y / \rho H$$
$$\frac{D\eta}{Dt} = -H\nabla \bullet \vec{V}$$

- **Consider the case of a North-South oriented coast line. In the case of a uniform, alongshore wind the linearized shallow water equations become**

$$\frac{\partial u}{\partial t} = -g\frac{\partial \eta}{\partial x} + fv$$
$$\frac{\partial v}{\partial t} = -fu + \tau_y / \rho H$$
$$\frac{\partial \eta}{\partial t} = -H\frac{\partial u}{\partial x}$$

  - **These equations can be combined into a single equation for *u*, which is**

$$\frac{\partial^2 u}{\partial t^2} - gH\frac{\partial^2 u}{\partial x^2} + f^2 u = \frac{f\tau_y}{\rho H} \tag{1}$$

- **The solution to equation (1) has a *transient* part that consists of inertia-gravity waves which propagate away as the ocean adjusts to geostrophic balance, and a *steady* solution that is in geostrophic balance.**

- **We can analyze the steady solution by ignoring the time derivative. This yields**

$$\frac{d^2u}{dx^2} - \frac{f^2}{gH}u = -\frac{f\tau_y}{\rho g H^2}.$$

- **From the definition of the radius of deformation**

$$\lambda \equiv \frac{c}{f} = \frac{\sqrt{gH}}{f}$$

the equation becomes

$$\frac{d^2u}{dx^2} - \frac{1}{\lambda^2}u = -\frac{f\tau_y}{\rho g H^2}. \qquad \textbf{(2)}$$

which is a nonhomogeneous, second order ODE with constant coefficients.

- **The solution to (2) consists of the solution to the homogeneous equation, plus a particular solution for the nonhomogeneous part. This is**

$$u(x) = A\exp(x/\lambda) + B\exp(-x/\lambda) + \tau_y / f\rho H.$$

- **The constants *A* and *B* are found from the boundary conditions.**
  - **In order for *u*(*x*) to be bounded as x goes to infinity, the coefficient *A* must be equal to zero.**
  - **In order for *u*(0) to be zero, the coefficient *B* must equal $-\tau_y/f\rho H$, so that**

$$u(x) = \frac{\tau_y}{f\rho H}\left[1 - \exp(-x/\lambda)\right]. \qquad \textbf{(3)}$$

- **From the continuity equation and equation (3) we have**

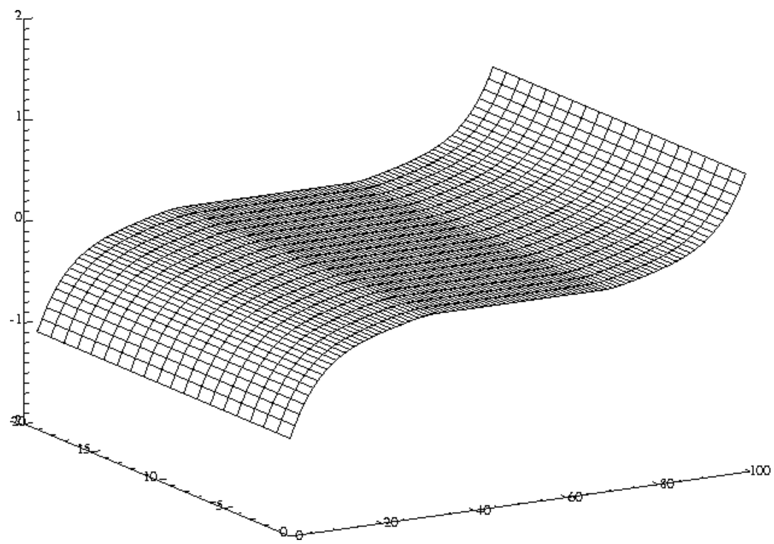
$$\frac{\partial \eta}{\partial t} = -H\frac{\partial u}{\partial x} = -H\frac{\tau_y}{\lambda f\rho H}\exp(-x/\lambda) = -\frac{\tau_y}{\rho\sqrt{gH}}\exp(-x/\lambda)$$

so that

$$\eta(x,t) = -\frac{\tau_y}{\rho\sqrt{gH}}\exp(-x/\lambda)t \qquad \textbf{(4)}$$

- **Equation (4) shows that the sea surface heights near a north-south oriented western boundary will increase linearly with time if the flow is from the north (τ_y < 0) and will decrease linearly with time if the flow is from the south (τ_y > 0).**
- **The sea surface height perturbation will decay exponentially away from the coast, with an e-folding scale of (you guessed it) the radius of deformation, λ.**

- **Notice also in equation (4) that the magnitude of the storm surge scales linearly with wind stress, and inversely with the square root of the depth.**
- **The figure below shows a plot of sea surface heights for a uniform southerly wind over an ocean basin. This figure was created from a numerical model using the shallow water equations.**



STORM SURGE, KELVIN WAVES, AND CONTINENTAL SHELF WAVES

- **Storm surge is often associated with the wind stress from a transient low pressure system, such as a tropical or extratropical cyclone. In this case the wind stress is not uniform.**
- **A nonuniform wind stress will generate a local storm surge. However, this local storm surge may then propagate along the coast as a forced Kelvin wave, or as a forced, continental shelf wave.**
- **Kelvin waves are long waves that travel parallel to the coast with the boundary to the right of propagation. They are nondispersive, and propagate at the shallow-water gravity wave speed.**
- **The wind stress may also cause another type of wave phenomenon, the continental shelf wave.**
- **Continental shelf waves owe their existence to the conservation of potential vorticity**

$$\frac{D}{Dt}\left(\frac{\varsigma + f}{H}\right) = 0 \qquad \textbf{(5)}$$

- **In much the same manner that a change in the Coriolis parameter leads to Rossby waves, a change in depth leads to continental shelf waves.**
- **If equation (5) is expanded out it becomes**

$$\frac{D\varsigma}{Dt} + \beta v - \frac{1}{H}(\varsigma + f)\vec{V} \bullet \nabla H = 0.$$

- **If this equation is linearized, *f* is assumed constant, there is mean flow, and *f* is much larger than ζ, it becomes**

$$\frac{\partial \varsigma}{\partial t} = \frac{f_0}{H}\left(u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right).$$

If the ocean bottom is assumed to slope only in the *y* direction then we get

$$\frac{\partial \varsigma}{\partial t} = \frac{f_0}{H} v \frac{\partial H}{\partial y}.$$

In terms of streamfunction this is

$$\frac{\partial}{\partial t}\nabla^2 \psi = \frac{f_0}{H}\frac{\partial H}{\partial y}\frac{\partial \psi}{\partial x}.$$

Substituting

$$\psi(x, y, t) = Ae^{i(kx+ly-\omega t)}$$

the following dispersion relation is found,

$$\omega = \frac{\left(\frac{1}{H}\frac{dH}{dy}\right)k}{k^2 + l^2}.$$

- **The dispersion relation for shelf waves is eerily similar to that for Rossby waves,**

$$\omega = \frac{-\beta k}{k^2 + l^2}.$$

- **Shelf waves are very similar to Rossby waves, except instead of requiring a non-constant Coriolis parameter, they require a non-constant depth.**
- **Shelf waves travel with the shore to the right, like Kelvin waves.**
- **Unlike Kelvin waves, shelf waves are dispersive.**