# ESCI 343 – Atmospheric Dynamics II
## Lesson 8 – Wave Modes

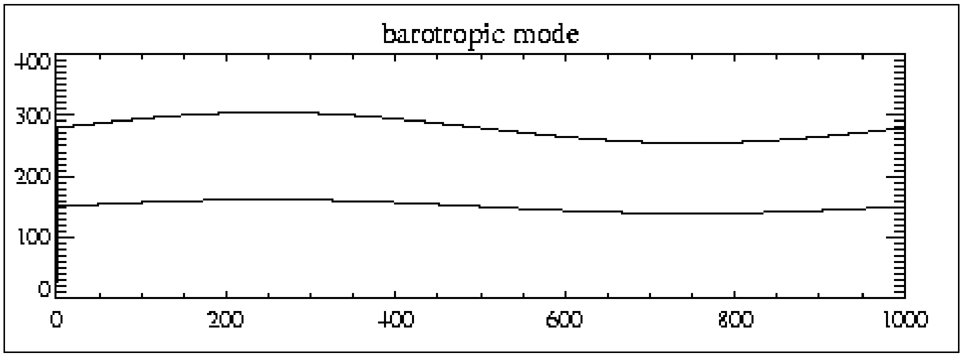
**References:** *Atmosphere-Ocean Dynamics*, A.E. Gill

## GENERAL

- Wave motions are important for describing equatorial adjustment and storm surge.
- Wave motions that are important are
  - Kelvin waves
  - Rossby waves
- The ocean can often be approximated as a two-layer fluid, with the two-layers being the mixed layer and the deep ocean. The thermocline separates these two layer.
- Wave motions in a two-layer fluid are more complex than in a single layer.
- We will first consider shallow-water gravity waves in a two-layer fluid, and then extend these concepts to Kelvin and Rossby waves.

## BAROTROPIC VERSUS BAROCLINIC MODES

- In a two-layer fluid gravity waves can exist on either the interface at the top of the upper fluid, or on the interface between the two fluids.
- We will denote the depth of the lower fluid by $H_1$, and the depth of the upper fluid by $H_2$. The total depth of the fluid is then $H = H_1 + H_2$.
- The waves on the top and on the interface are not independent. Instead, they are locked together in two possible *modes*.
- *Barotropic mode* – In the barotropic mode the two surfaces move in phase with one another as shown below



- ○ **The phase speed of the barotropic mode is**

$$c^2 = g\left(H_1 + H_2\right) = gH \tag{1}$$

**where *H* is the total depth of the fluid. Thus, the barotropic mode behaves just like an ordinary shallow-water gravity wave.**

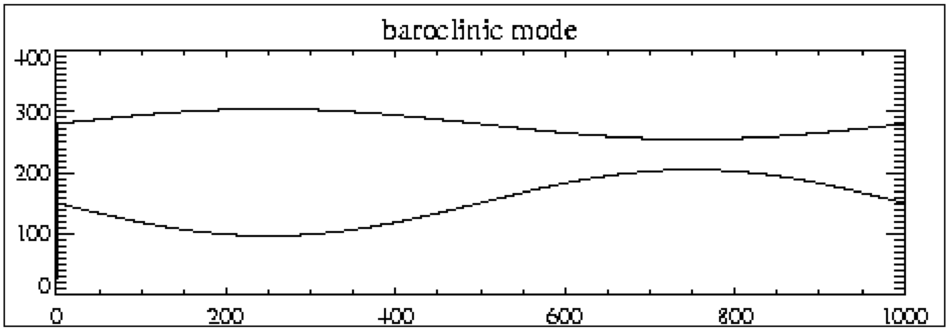
- ○ **The ratio of the amplitudes of the waves on the two surfaces are given by**

$$A_2/A_1 = \left(H_1 + H_2\right)/H_1\,, \tag{2}$$

**where $A_2$ is the amplitude of the upper wave and $A_1$ is the amplitude of the lower wave.**

- ○ **The upper wave is larger than the lower wave.**

- ● ***Baroclinic mode* – The baroclinic mode is more interesting, as the two waves have opposite phase as shown below**



- ○ **The phase speed of the baroclinic mode is**

$$c^2 = g'\frac{H_1H_2}{H} \tag{3}$$

**where**

$$g' = \frac{\rho_1 - \rho_2}{\rho_1}g \tag{4}$$

**and is called *reduced gravity* (it is always less than *g*), and *H* is the total depth of the fluid.**

- ○ ***The baroclinic mode is slower than the barotropic mode*.**
- ○ **The ratio of the amplitudes of the two waves is**

$$\frac{A_2}{A_1} \cong -\frac{g'}{g}\frac{H_1}{H} \tag{5}$$

**so that *the upper wave is significantly smaller than the lower wave*.**

- ***Equivalent Depth* – We can manipulate the equation for the phase speed of the baroclinic mode to get**

$$c^2 = g'\frac{H_1H_2}{H} = g\left(\frac{g'H_1H_2}{gH}\right) = gH_e \tag{6}$$

**where**

$$H_e = \frac{g'H_1H_2}{gH} \tag{7}$$

**is called the *equivalent depth*.**

  - **The equivalent depth is called such because it is the depth of a fluid whose barotropic mode has the same speed as the baroclinic mode of the original two layer fluid.**

**MULTIPLE LAYERED FLUIDS**

- **A three-layer fluid would have a barotropic mode and two baroclinic modes.**
- **A four-layer fluid would have a barotropic mode and three baroclinic modes.**
- **An *n*-layered fluid would have a barotropic mode and *n* – 1 baroclinic modes.**
- **A continuously stratified fluid essentially has an infinite number of layers, and so has an infinite number of baroclinic modes.**
  - **Not all of these modes would be excited.**
  - **If we can identify which modes are important, and calculate their equivalent depths, we can simplify looking at waves in a continuously stratified fluid.**

**RADIUS OF DEFORMATION**

- **In middle latitudes on the *f*-plane the atmosphere and oceans each tend to be in geostrophic and hydrostatic balance.**
- **When conditions "push" the atmosphere or ocean out of balance they adjust by generating inertia-gravity waves (gravity waves that are of long enough wavelengths to be influenced by the rotation of the Earth).**
- **The final state after adjustment is dependent on the radius of deformation, given by**

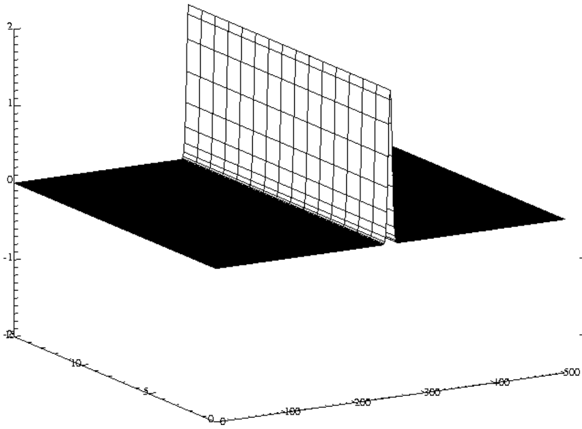
$$\lambda = c/f\,, \tag{8}$$

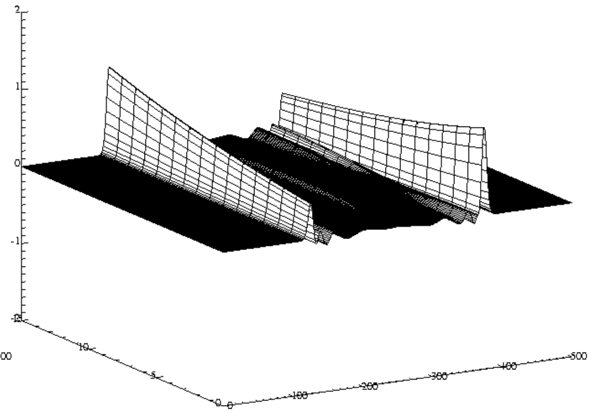
where $c$ is the group speed of a gravity wave ($c = \sqrt{gH}$) and $f$ is the Coriolis parameter.

- For disturbances that are larger than the radius of deformation ($L >> 2\pi\lambda$), the velocity adjusts to the mass (height) field.
- For disturbances that are smaller ($L << 2\pi\lambda$) than the radius of deformation the mass field adjusts to the velocity.
- For disturbances that are of the order of the radius of deformation ($L \sim 2\pi\lambda$), the height and velocity fields undergo mutual adjustment.
- *NOTE: Each mode of a multi-layer fluid has its own radius of deformation.*
- *The baroclinic radii of deformation are much smaller than the barotropic radius of deformation.*

KELVIN WAVES

- Along a rigid lateral boundary, on a rotating planet, a special type of wave called a Kelvin wave is possible (see initial and later times below).



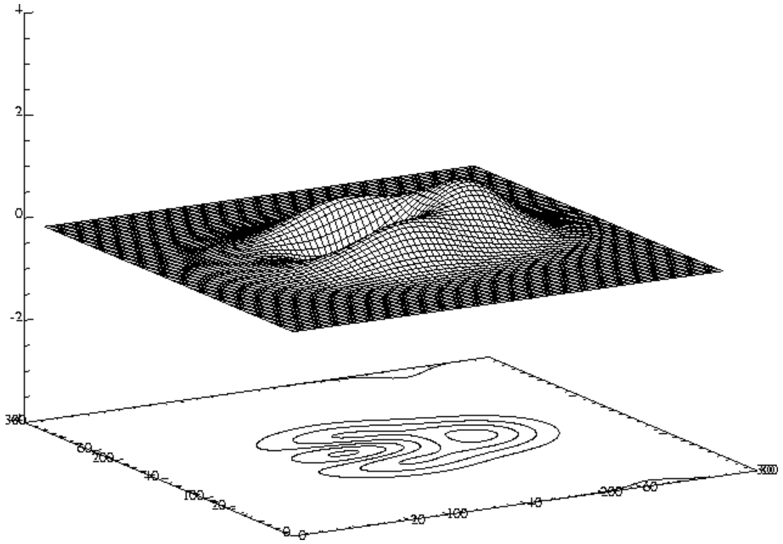


- Kelvin waves propagate with the boundary on the right in the Northern Hemisphere (on the left in the Southern Hemisphere).
  - The Kelvin wave is sometimes described as *leaning* on the boundary.
- The amplitude of the Kelvin wave decreases exponentially with distance from the boundary.
  - The *e*-folding scale of the decay is the radius of deformation.
- Kelvin waves are non-dispersive, and travel at the speed of a gravity wave.
- Kelvin waves are observed in the ocean along coastlines.

  - **It may be possible to also have Kelvin waves in the atmosphere, such as at the top of the boundary layer, but they are difficult to observe.**

**WAVES IN THE TROPICAL OCEANS**

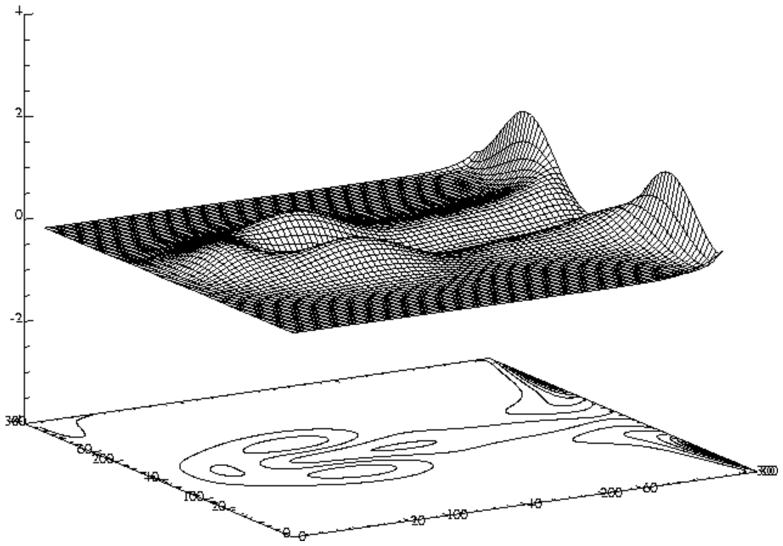
- **Two types of waves that play a key role in the adjustment of the tropical oceans to changes in wind stress are *equatorially trapped Kelvin wave*, and *equatorial Rossby waves*.**
- **Equatorially trapped Kelvin waves are symmetric with the Equator (see figure below).**



  - **The equatorially trapped Kelvin wave is non-dispersive, and travels eastward at the same velocity as a shallow-water gravity wave**

$$c = \sqrt{gH} \, .$$

  - **The equatorial Kelvin wave is similar to the regular Kelvin wave in that it leans against a boundary. However, instead of physical boundary, it is a *dynamic boundary* caused by the change in sign of *f* across the Equator.**
- **When the equatorially trapped Kelvin wave impacts the eastern boundary of the basin it reflects as two coastally trapped Kelvin waves, and a long equatorial Rossby wave (see figure below).**



- **Rossby waves have a westward phase velocity.**
  - **Rossby waves are dispersive, with a westward group velocity for long waves, and an eastward group velocity for short waves.**
  - **When a long Rossby wave impacts on the western boundary it reflects as short Rossby waves.**
  - **The short Rossby waves attenuate quickly, so that energy tends to collect along the western boundary.**
- **Kelvin and Rossby waves can also occur in two-layered or multi-layered fluids, and so can have barotropic as well as baroclinic modes.**