ESCI 485 – Air/sea Interaction
Lesson 5 – Oceanic Boundary Layer

References: ***Descriptive Physical Oceanography*, Pickard and Emery**
***Introductory Dynamical Oceanography*, Pond and Pickard**
***Principles of Ocean Physics*, Apel**

DENSITY AND SALINITY

- **The density of the ocean is determined by two factors: temperature and salinity.**
- **Density increases with decreasing temperature**
- **Density increases with increasing salinity.**
- **Usually the temperature is the dominant factor in controlling density. Therefore, a warmer temperature above cooler temperatures usually is a sign of stability.**
    - **However, in areas with large salinity gradients it is possible to have colder fresh water over warmer, salty water and still be stable.**

MIXED LAYER

- **The ocean can be divided into three layers**
    - ***Mixed* (or *surface*) layer**
    - ***Thermocline***
    - ***Deep layer***
- **The mixed layer is akin to the atmospheres planetary boundary layer.**
- **The mixed layer gets its name from the fact that it tends to be well mixed, with the temperature being nearly isothermal with depth.**
- **The depth of the mixed layer varies with location and season. Typical ranges are from 25 to 500 meters.**
- **The depth is determined primarily by how rough the seas are. The rougher the seas, the deeper the mixing.**
    - **Since seas are generally rougher in winter, the mixed layer depth is usually deeper in winter than in summer.**

THERMOCLINE

- **At the bottom of the mixed layer is the beginning of the *thermocline*.**
- **The thermocline is characterized by a decrease in temperature with depth.**
- **The thermocline is a very stable layer. Because of this, vertical mixing in the ocean at depths below the mixed layer is very slow.**
- **Because the ocean typically has a strong thermocline that inhibits mixing between the mixed layer and the deep layer, it is sometimes conceptually and mathematically convenient to model the ocean as a two-layer fluid.**

EQUATIONS GOVERNING OCEAN DYNAMICS

- **The equations that govern the dynamics of the ocean are nearly identical to those for the atmosphere. They are the three momentum equations, the continuity equation, the thermodynamic energy equation, and an equation of state. There is no equation for the continuity of water vapor, but there is an equation for the continuity of salinity.**
- **Like the atmosphere, for large-scale flow the ocean can be considered to be in hydrostatic balance.**
- **The ocean can also usually be considered as incompressible (even more so than the atmosphere).**
- **Except in a thin layer right near a boundary, viscous forces can be neglected.**
- **The three momentum equations and the continuity equation for the ocean are therefore**

$$
\begin{aligned}
\frac{Du}{Dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z} \\
\frac{Dv}{Dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + \frac{1}{\rho}\frac{\partial \tau_y}{\partial z} \\
\frac{\partial p}{\partial z} &= -\rho g \\
\frac{\partial u}{\partial x} &+ \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\end{aligned}
\qquad \textbf{(1)}
$$

where the stress terms are due to the Reynolds, or vertical turbulent momentum fluxes.

THE EKMAN SPIRAL

- **An oceanographer named Nansen, around 1898, came up with a qualitative argument as to why icebergs tend to blow at an angle to the right of the wind. His argument was based on a balance of the wind stress (the wind force on the iceberg), the Coriolis force, and friction.**
- **A few years later, Nansen's assistant (Ekman) formulated a quantitative argument.**
- **Ekman began with the two momentum equations above. However, he assumed no pressure gradient force, and also assumed steady motion (so the time derivatives become zero). This leaves the following two equations**

$$fv + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z} = 0$$
$$fu - \frac{1}{\rho}\frac{\partial \tau_y}{\partial z} = 0 \qquad \textbf{(2)}$$

- **The Reynolds stresses were parameterized in terms of an eddy viscosity, *K*, such that**

$$\tau_x = \rho K \frac{\partial u}{\partial z}$$
$$\tau_y = \rho K \frac{\partial v}{\partial z} \qquad \textbf{(3)}$$

so that the equations governing the flow are

$$\frac{d^2 u}{dz^2} + \frac{f}{K} v = 0 \qquad \textbf{(4)}$$

$$\frac{d^2 v}{dz^2} - \frac{f}{K} u = 0 \qquad \textbf{(5)}$$

(we've changed the partial derivatives to regular derivatives since *z* is the only independent variable).

- **Equations (4) and (5) are a set of coupled, 2nd-order ODE's. We can resort to a little trick to solve them. We define a complex velocity so that**

$$w = u + iv\,. \qquad \textbf{(6)}$$

If we multiply Eq. (5) by *i* and then add it to Eq. (4) we get

$$\frac{d^2(u+iv)}{dz^2}+\frac{f}{K}(v-iu)=0. \qquad \textbf{(7)}$$

Manipulating the complex number we can show

$$v-iu=i(v/i-u)=i(-iv-u)=-i(u+iv)$$

so that our equation becomes

$$\frac{d^2(u+iv)}{dz^2}-i\frac{f}{K}(u+iv)=0$$

or

$$\frac{d^2w}{dz^2}-i\frac{f}{K}w=0. \qquad \textbf{(8)}$$

- **Our trick turned the coupled set of ODE's into a single ODE of a complex variable. But, a single ODE is easier to solve than a system of ODE's.**

- **The general solution to Eq. (8) is**

$$w(z)=A\exp\left(z\sqrt{if/K}\right)+B\exp\left(-z\sqrt{if/K}\right). \qquad \textbf{(9)}$$

- **By another identity of complex numbers we can show that** $\sqrt{i}=\frac{i+1}{\sqrt{2}}$**, so that we can write the solution as**

$$w(z)=A\exp(\gamma z)\exp(i\gamma z)+B\exp(-\gamma z)\exp(-i\gamma z) \qquad \textbf{(10)}$$

where

$$\gamma\equiv\sqrt{f/2K} \qquad \textbf{(11)}$$

- **To find the constants *A* and *B* we have to apply the boundary conditions at the surface and at depth.**
  - **We require the velocity to vanish as depth increases (*z* becomes more negative). This implies that *B* = 0.**
  - **At the surface we require that $w = W_0$, the surface current. This implies that $A = W_0$.**

- **The solution to the equation is then**

$$w(z)=W_0\exp(\gamma z)\exp(i\gamma z). \qquad \textbf{(12)}$$

- **Using Euler's formula we can write Eq. (12) as**

$$u+iv=(U_0+iV_0)\exp(\gamma z)[\cos(\gamma z)+i\sin(\gamma z)]. \qquad \textbf{(13)}$$

  - **Separating the real and imaginary parts, and writing each separately, we get**

$$
\begin{aligned}
u(z) &= \exp(\gamma z)[U_0 \cos(\gamma z) - V_0 \sin(\gamma z)] \\
v(z) &= \exp(\gamma z)[V_0 \cos(\gamma z) + U_0 \sin(\gamma z)] \quad .
\end{aligned} \tag{14}
$$

- **The characteristics of this current profile are more easily seen if we assume the surface current is strictly zonal ($V_0 = 0$). Then Eqs. (14) become**

$$
\begin{aligned}
u(z) &= U_0 \exp(\gamma z)\cos(\gamma z) \\
v(z) &= U_0 \exp(\gamma z)\sin(\gamma z) \quad .
\end{aligned} \tag{15}
$$

- **If the current is plotted on a hodograph it traces a decaying clockwise spiral with depth. This is known as the *Ekman spiral*.**
- **The depth of the Ekman layer is taken to be that point at which the current has decayed by a fraction of e^{-1} (the *e*-folding scale). Therefore, the depth of the Ekman layer is**

$$
D_E = \frac{1}{\gamma} = \sqrt{\frac{2K}{f}} \, . \tag{16}
$$

THE SURFACE CURRENT

- **We still haven't explained the fact that the surface current is to the right of the wind speed. To do this we have to look at another boundary condition at the surface.**
- **At the surface the stress must be continuous (i.e., the stress in the air must equal that in the water).**
- **The stress in the air at the surface is just the wind stress, and so is in the direction of the surface wind.**
- **The stress in the water at the surface is that due to the Reynolds stresses, and is given by**

$$
\begin{aligned}
\tau_x &= \rho K \left.\frac{\partial u}{\partial z}\right|_{z=0} \\
\tau_y &= \rho K \left.\frac{\partial v}{\partial z}\right|_{z=0} \quad .
\end{aligned} \tag{17}
$$

- **Taking the case of our purely zonal current, and applying the boundary conditions above we get**

$$\tau_x = \rho K \frac{d}{dz}(U_0 \exp(\gamma z)\cos(\gamma z)\Big|_{z=0} = \rho K \gamma U_0$$
$$\tau_y = \rho K \frac{d}{dz}(U_0 \exp(\gamma z)\sin(\gamma z)\Big|_{z=0} = \rho K \gamma U_0 \quad . \qquad \textbf{(18)}$$

This gives us two important pieces of information.

- **It tells us that** ***for our purely zonal current that there had to be a non-zonal component of wind stress*****. So, the current isn't flowing in the direction of the wind, but at an angle to the wind.**
- **Since the magnitudes of the *x* and *y*-components of the wind stress are equal, this means that** ***the surface current should be flowing at exactly 45° to the right of the surface wind.*** **This is a quantitative description of the effect that Nansen observed!**
- **It tells us the speed of the surface current in terms of the magnitude of the wind stress, since**

$$|\tau| = \sqrt{2}\rho K \gamma U_0 = \rho U_0 \sqrt{Kf}$$

or

$$U_0 = \frac{|\tau|}{\rho\sqrt{Kf}} \, . \qquad \textbf{(19)}$$

EKMAN TRANSPORT AND EKMAN PUMPING

- **The net transport of water in the Ekman layer can be found by integrating the Ekman equations through the depth of the layer,**

$$M_x = \int_{-\infty}^{0} \rho u(z) dz = \rho U_0 \int_{-\infty}^{0} \exp(\gamma z)\cos(\gamma z)\, dz = \rho U_0 / 2\gamma$$
$$M_y = \int_{-\infty}^{0} \rho v(z) dz = \rho U_0 \int_{-\infty}^{0} \exp(\gamma z)\sin(\gamma z)\, dz = -\rho U_0 / 2\gamma \quad . \qquad \textbf{(20)}$$

- **This result shows that the net transport is directed at 45° to the right of the surface current.**
- **Since the surface current itself is directed at 45° to the right of the surface wind we have shown that** ***the net transport in the oceanic Ekman layer is directed at 90° to the right of the surface wind*****.**
- **The total Ekman transport is**

$$M = \sqrt{M_x^2 + M_y^2} = \rho U_0 \big/ \gamma\sqrt{2} = |\tau| \big/ f \,. \qquad \textbf{(21)}$$

- Since the Ekman transport is directed at 90° to the right of the wind stress, we can write the following vector equation

$$\vec{M} = -\frac{1}{f}\hat{k} \times \vec{\tau} \,. \qquad \textbf{(22)}$$

EKMAN TRANSPORT AND OCEAN CIRCULATION

- **Ekman transport has important implications for the ocean circulation.**
  - **The anticylonically rotating wind-driven gyres in the ocean basins will have a net Ekman transport toward the center of the gyre.**
  - **The surface convergence in the center of the gyre results in elevated sea level heights in the gyre's center.**
  - **The surface convergence also pushes the colder, deeper water to even greater depths.**
  - **The surface convergence must be compensated by downward vertical motion (called *downwelling*).**
  - **This entire process is sometimes referred to as *Ekman pumping*.**
  - **Ekman pumping results in a secondary circulation superimposed on the gyre, with convergence and downward motion in the middle of the gyre, and divergence deeper in the gyre.**
- **The vertical velocities associated with this downwelling can be found by integrating the continuity equation through the depth of the Ekman layer**

$$\int_{-D_e}^{0} \frac{\partial w}{\partial z} dz = -\int_{-D_e}^{0} \nabla \bullet \vec{V} dz = -\frac{1}{\rho} \nabla \bullet \vec{M} \,.$$

From Eq. (22) we have

$$\int_{-D_e}^{0} \frac{\partial w}{\partial z} dz = -\frac{1}{\rho} \nabla \bullet \frac{1}{f} \hat{k} \times \vec{\tau} = -\frac{1}{\rho f} \nabla \times \vec{\tau} \,.$$

The integral on the left-hand-side is

$$\int_{-D_e}^{0} \frac{\partial w}{\partial z} dz = w(0) - w_E = -w_E$$

(the vertical velocity at the surface is zero). Therefore, we get the result

$$w_E = \frac{1}{\rho f} \nabla \times \vec{\tau} \,. \qquad \textbf{(23)}$$

- **This result says that *the vertical velocity at the bottom of the Ekman layer is proportional to the curl of the wind stress*.**
- **A cyclonic wind stress will give upwelling, and an anticyclonic wind stress will give downwelling.**

- **Ekman transport may be important in the response of the ocean to individual storms. The wind stress from a cyclonically rotating storm will induce Ekman transport away from the center of the storm, and can result in local upwelling near the center of the circulation.**

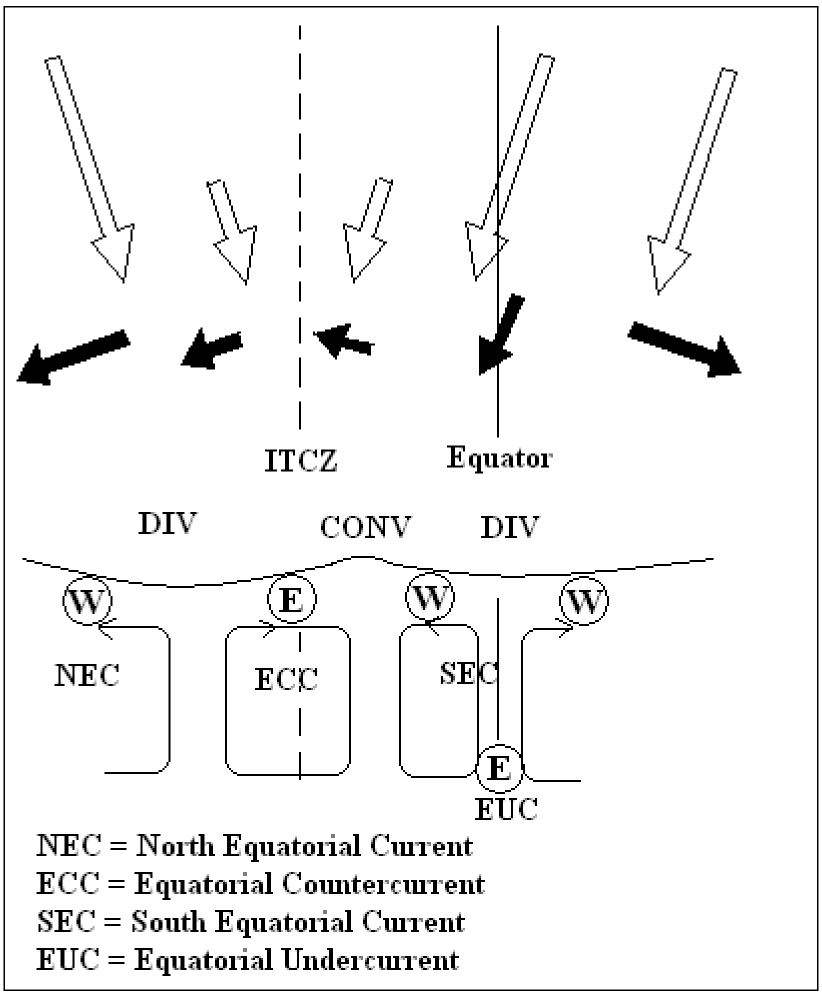
EKMAN TRANSPORT AND THE SEASONAL CLIMATE OF THE WEST COAST

- **Ekman transport also has dramatic and important effects on the climate of the West Coasts of North and South America.**
  - **Along the west coast of North America in the spring and summer the Pacific High moves further offshore. The prevailing winds during this period are from the northwest to north-north west.**
  - **Ekman transport is therefore directed off-shore, and pulls surface water away from the coast.**
  - **This horizontal divergence at the surface is compensated by upwelling, which brings colder water from the deep ocean up to the surface.**
  - **This upwelling causes the waters off the west coast of North America to be colder in the spring and summer, than at other times of the year, and explains why the coastal climate of Central and Northern California, Oregon, and Washington is so cool and often foggy in the summer.**
  - **In the fall and winter the Pacific high is farther to the north and west, and the upwelling is not present.**
  - **It is the upwelling associated with the Ekman transport that is the reason the Pacific Coast often has very nice weather in the fall, but is usually cold and foggy in spring and summer.**

- **This same phenomena is observed off of the West Coast of South America also.**

EQUATORIAL CURRENTS

- **The surface currents in the Pacific and Atlantic have a similar structure, and can be explained at least in part by convergence and divergence associated with Ekman transport.**



- **The diagram above shows the air flow (open arrows) and the resultant Ekman transport (dark arrows).**
- **The "DIV" and "CONV" denote regions of divergence and convergence in the Ekman transport.**
  - **Regions of divergence will result in a lowering of the sea-surface, while regions of convergence will raise the sea surface.**

- **Other than within a degree or so of the Equator, the ocean flow will be parallel to the sea-surface contours with low heights to the left in the Northern Hemisphere, and to the right in the Southern Hemisphere.**
  - **The ocean currents and their directions are indicated by the "W" and "E" annotations, with "W" indicating a westward current, while "E" indicates an eastward current.**
- **The resultant surface currents are the**
  - **North Equatorial Current – Westward flowing**
  - **Equatorial Counter Current – Eastward flowing, more-or-less aligned with the ITCZ.**
  - **South Equatorial Current – Westward flowing, and in both hemispheres.**
- **There is also an Eastward flowing Equatorial Undercurrent that more-or-less flows along the Equator at depth.**
- **The equatorial currents in the tropical Indian Ocean differ in that since the atmospheric flow switches directions seasonally due to the monsoon, so do the currents.**