

**ESCI 485 – Air/sea Interaction**  
**Lesson 2 – Turbulence**  
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**References:** *Air-sea Interaction: Laws and Mechanisms*, Csanady  
*An Introduction to Dynamic Meteorology (3<sup>rd</sup> edition)*, J.R. Holton  
*An Introduction to Boundary Layer Meteorology*, R.B. Stull  
*Structure of the Atmospheric Boundary Layer*, Sorbjan

**TURBULENT KINETIC ENERGY**

- **Turbulent kinetic energy (TKE) is the kinetic energy (per unit mass) associated with turbulent eddies.**
- **Mathematically it is defined as**

$$e = \frac{1}{2} (\overline{u'^2 + v'^2 + w'^2}). \quad (1)$$

- **Turbulent kinetic energy is primarily transferred from larger scales to smaller scales.**
  - **At the large scales the turbulence is generated via mechanical means (through sheared flows) or via buoyancy.**
  - **Energy is *dissipated* at the very smallest scales due to molecular friction. This flow of turbulent energy from large scales to small scales is known as the *energy cascade*.**
  - **The energy cascade is summed up very aptly in the verse,**

*Big whorls have little whorls,  
Which feed on their velocity,  
And little whorls have lesser whorls,  
And so on to viscosity.*

- L. F. Richardson

- **An equation for the budget of turbulent kinetic energy (TKE) is given symbolically as**

$$\frac{\overline{D}e}{Dt} = MP + BP + TR - \epsilon. \quad (2)$$

- *MP* represents the mechanical production of turbulence.
- *BP* represents buoyant production.
- *TR* represents turbulent transport and redistribution by pressure perturbations.
- $\epsilon$  represents molecular dissipation.
- **Mechanical production (MP)** – The mechanical production term has the form

$$MP = -\overline{u'w'} \frac{\partial \bar{u}}{\partial z}. \quad (3)$$

- Mechanical production occurs because of dynamic instabilities caused by sheared flow. The stronger the shear of the mean flow, the more turbulence will be produced.
- This term can also be negative, and represent a loss of *TKE*.
- **Buoyant production (BP)** – The buoyant production term has the form

$$BP = \overline{w'b'}, \quad (4)$$

where

$$b' = -g \frac{\rho'}{\rho_0} \quad (5)$$

and is the buoyancy perturbation.

- Buoyant production represents the production of turbulence due to thermals caused by heating of the surface (in the ocean, buoyant production would result from cooling of the surface).
- Like MP, the BP term is usually largest near the surface. In a stable layer this term will be negative and represent a loss of *TKE*.
- **Transport and redistribution (TR)** – This term has two components, and is written as

$$TR = -\left( \overline{u' \frac{\partial e}{\partial x}} + \overline{w' \frac{\partial e}{\partial z}} \right) - \frac{1}{\rho_0} \left( \overline{u' \frac{\partial p'}{\partial x}} + \overline{w' \frac{\partial p'}{\partial z}} \right). \quad (6)$$

- The first component represents the transport (or advection) of *TKE* by turbulent eddies.
- The second component represents the effects of pressure perturbations.

- **Dissipation** – The dissipation term has the form

$$\varepsilon = \nu \overline{\left[ \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial z} \right) (u' + w') \right]^2}. \quad (7)$$

- Dissipation represents the loss of energy due to molecular friction ( $\nu$  is the kinematic viscosity), and is always positive, so that in the *TKE* equation dissipation always represents a loss term.

### OTHER TURBULENCE CHARACTERIZATIONS

- **Velocity scale**

- One possible velocity scale is

$$u_m^2 = \overline{u'^2}. \quad (8)$$

- If the turbulence is isotropic, then  $w'$  is of the same order as  $u'$ , and Eq. (8) can be written as

$$u_m^2 = \left| \overline{u'w'} \right| \quad (9)$$

(the absolute value is needed to ensure that the velocity is real and not imaginary. This gives rise to the definition of *friction velocity*

$$u^* = \left( \left| \overline{u'w'} \right| \right)^{1/2}. \quad (10)$$

- **Eddy size**

- Eddy size is determined statistically
- Use a “two-point” correlation function

$$R_w(r) = \overline{w'(x)w'(x+r)}. \quad (11)$$

- The distance at which the velocity is uncorrelated ( $R = 0$ ) is taken to be the characteristic eddy size or length scale,  $l$ .
- The correlation function can also be defined in terms of horizontal velocities

$$R_u(r) = \overline{u'(x)u'(x+r)}. \quad (12)$$

- **Fourier spectra**

- **The Fourier transform pairs for a function of  $x$  are**

$$F(k) = \int_{-\infty}^{\infty} f(x) \exp(-ikx) dx$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k) \exp(ikx) dk \quad .$$
(13)

- **The Fourier transform of the correlation function  $R_u(r)$  gives the *spectral energy density* of the turbulence**

$$\phi(k) = \int_{-\infty}^{\infty} R_u(r) \exp(-ikr) dr .$$
(14)

- **The wave number at which the energy density peaks ( $k_p$ ) can be taken as a characteristic wave number of the turbulence, and from the relation**

$$\lambda_p = 2\pi/k_p$$
(15)

**characteristic wavelength of the turbulent eddies can be computed.**

- **Dissipation rate**

- **The rate at which turbulence dissipates kinetic energy is another means of characterizing turbulence.**
- **Dissipation rate is given the symbol ( $\epsilon$ ) and has units of  $\text{J s}^{-1} \text{kg}^{-1}$ .**
- **Dissipation converts kinetic energy into thermal energy, due to viscous effects.**
- **Dissipation rate has been shown to be proportional to**

$$\epsilon \propto u_m^3 / l .$$

### TAYLOR'S HYPOTHESIS

- **The correlation function  $R(r)$  is a spatial correlation. However, data are usually taken at a single point as a time series.**
- **We make use of Taylor's hypothesis in order to use the time series data to infer spatial properties of the turbulence.**
- **Taylor's hypothesis states that if  $u'/\bar{u} \ll 1$ , then  $x = Ut$ , where  $U$  is the mean velocity.**

- Using Taylor’s hypothesis we can do spectral analysis in frequency ( $\omega$ ), and then convert it to wave number via

$$k = \omega/U .$$

## TURBULENCE SPECTRA

- The energy spectra of turbulence is theorized to have some general characteristics.
- The lowest wave numbers are the energy containing range.
- The highest wave numbers are the dissipation range
  - In the dissipation range, dimensional analysis predicts that the spectral density should be a function of dissipation rate and wave number only, via

$$\phi(k) = a\varepsilon^{2/3} k^{-2/3} . \tag{16}$$

- If the proportionality constant is known, and if the spectral density is measured, then the dissipation rate can be inferred.
- In between the energy containing range and the dissipation range is the inertial subrange, where the energy is “just passing through” on its way to smaller scales.
  - In the inertial subrange, dimensional analysis predicts that the spectral density should be given by

$$\phi(k) = b\varepsilon^{2/3} k^{-5/3} . \tag{17}$$

- On a log-log plot, an idealized turbulence spectrum would look like

