ESCI 343 – Atmospheric Dynamics II
Lesson 15 – Barotropic and Baroclinic Instability

Reference: *Numerical Prediction and Dynamic Meteorology (2nd edition)*, G.J. Haltiner and R.T. Williams
An Introduction to Dynamic Meteorology (3rd edition), J.R. Holton
Dynamics of the Atmosphere: A Course in Theoretical Meteorology, W. Zdunkowski and A. Bott

HYDRODYNAMIC INSTABILITY

A flow is hydrodynamically unstable if a small perturbation in the flow grows spontaneously. Examples of hydrodynamic instability that we've already studied are buoyant instability and inertial instability. In both these cases an air parcel moved from its original position will continue to accelerate away from where it started, instead of oscillating around its original position.

One method of assessing whether or not a flow is stable or unstable is by assuming that the perturbation has a sinusoidal waveform such as

$$\psi' = Ae^{i(kx-\omega t)}$$

and determining under what circumstances the frequency will have an imaginary component. If the dispersion relation has an imaginary component such as

$$\omega = \omega_r + i\omega_i ,$$

then the perturbation will have the form

$$\psi' = Ae^{i(kx-\omega t)} = Ae^{i(kx-\omega_r t)}e^{k\omega_i t}$$

which grows exponentially in time (and is therefore unstable) if $\omega_i > 0$.

An example of hydrodynamic instability: Internal gravity waves with imaginary Brunt -Vaisala frequency

Recall that the phase speed for internal gravity waves is

$$\omega = \pm\frac{N\mathrm{K}_H}{\mathrm{K}} ,$$

where N is the Brunt-Vaisala frequency given by

$$N^2 = -\frac{g}{\bar{\rho}}\frac{d\bar{\rho}}{dz} - \frac{g^2}{c_s^2} .$$

If N is real then the fluid is stable, and a parcel disturbed vertically from rest would oscillate about its original position. However, if N is imaginary then we know a parcel will be unstable, and if perturbed from rest it will accelerate away from its original position. This can also be seen from the dispersion relation, since N will be imaginary, and hence ω will have an imaginary component.

BAROTROPIC INSTABILITY

One form of hydrodynamic instability that can occur in the atmosphere is barotropic instability. The derivation of the condition for barotropic instability is beyond the scope of this course. But, the condition for barotropic instability involves the horizontal shear of the mean wind. The necessary condition for barotropic instability to occur is that, somewhere within the flow, the following condition must be true:

$$\frac{d^2\overline{u}}{dy^2} - \beta = 0 . \qquad \textbf{(1)}$$

This means that for barotropic instability to occur that the second derivative of the mean zonal wind must be equal to β somewhere in the flow. Condition (1) can also be written as

$$\frac{d}{dy}\left(\frac{d\overline{u}}{dy} - f\right) = 0 \qquad \textbf{(2)}$$

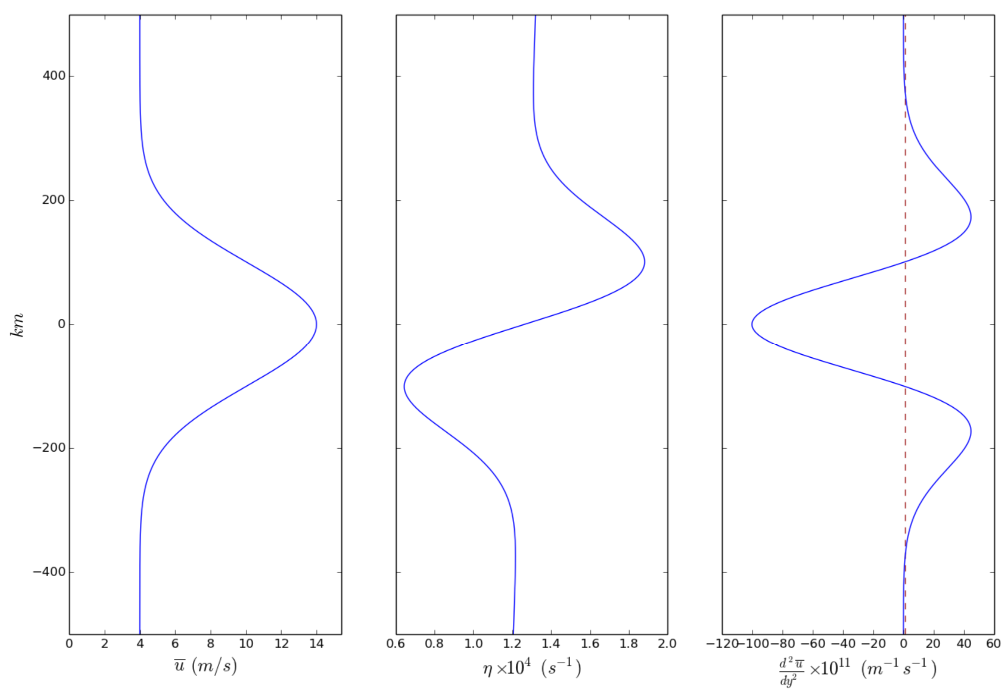
or

$$\frac{d\eta}{dy} = 0 . \qquad \textbf{(3)}$$

We can interpret this to mean that the absolute vorticity must have a minimum or maximum value somewhere in the flow in order for barotropic instability to occur.

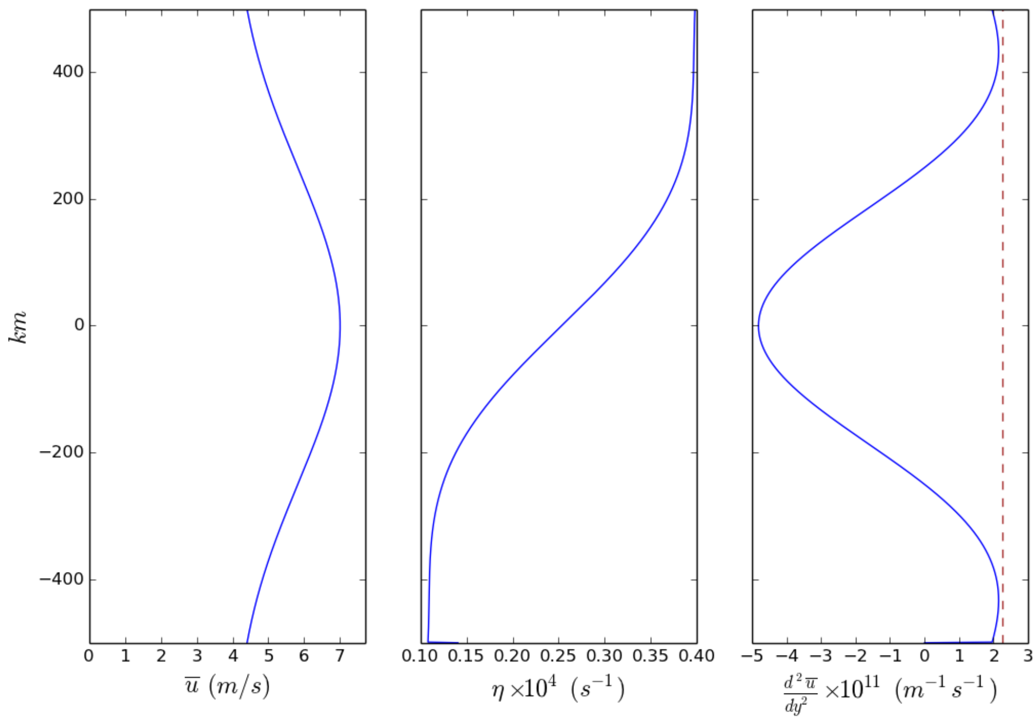
BAROTROPIC INSTABILITY IN A WESTERLY JET STREAM

Barotropic instability is dependent upon horizontal shear of the mean flow. To examine if barotropic instability is possible, the horizontal profile of the absolute vorticity must be examined. The plot below shows the zonal velocity, absolute vorticity, and the second derivative of the velocity for an idealized westerly jet stream on the beta plane. The dashed line on the third diagram is the value of beta.



There are absolute vorticity minima and maxima on both flanks of the jet, near the locations of the inflection points in the velocity profile. Thus, the condition for barotropic instability is met in these two regions.

However, the presence of an inflection point does not automatically mean that there is a minimum or maximum in the absolute vorticity. If beta is large compared to the second derivative of the velocity, such as for a broad, weak jet at low latitudes, as shown below, then there will not be any maxima or minima in vorticity, even though there are inflection points in the velocity profile. Thus, beta acts as stabilizing influence against barotropic instability.



ENERGETICS OF BAROTROPIC DISTURBANCES

Barotropic disturbances derive their energy from the mean flow. Energy considerations show that ***for a barotropic disturbance to grow it must tilt opposite to*** $d\overline{u}/dy$.[1] Since midlatitude disturbances tend to tilt in the same direction as $d\overline{u}/dy$, they actually lose energy back to the mean flow due to barotropic instability. Thus, barotropic instability is not a viable way for midlatitude disturbances to form and grow. However, interestingly enough, ***since midlatitude disturbance decay due to barotropic instability, they give up energy to the mean flow and help maintain the mean flow against friction***. Thus, barotropic instability is somewhat important for the maintenance of the mean flow in the midlatitudes.

BAROCLINIC INSTABILITY IN A TWO-LAYER FLUID

[1] See Haltiner and Williams, pp.74-75.

Since barotropic instability is not a viable option for the formation of midlatitude cyclones, then another mechanism must be invoked. This mechanism is ***baroclinic instability***. For baroclinic instability it is the ***vertical shear***, rather than the horizontal shear, that is important.

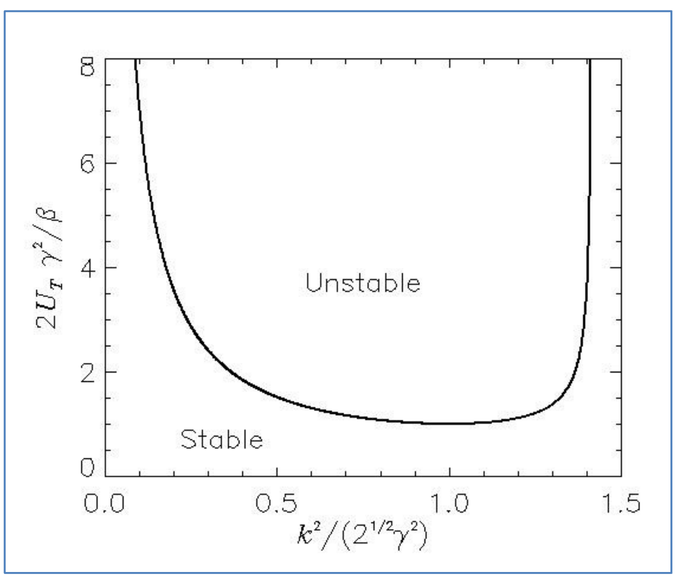
Baroclinic instability is often studied for the simple case of a two-layer fluid, for which waves are unstable if the following condition is true:

$$\delta = \frac{\beta^2\gamma^4}{k^4\left(k^2+2\gamma^2\right)^2} - \frac{U_T^2\left(2\gamma^2-k^2\right)}{\left(k^2+2\gamma^2\right)} < 0 . \qquad \textbf{(4)}$$

In this expression, U_T is the vertical wind shear parameter (equal to half the difference in U between the two layers, and γ is inversely proportional to static stability (large stability means small γ). This expression points out the importance of vertical wind shear on baroclinic instability. There is also a wavelength dependence for baroclinic instability, which is best illustrated by writing Eq. (4) in terms of the dimensionless variables

$$\tilde{U} = \frac{2U_T\gamma^2}{\beta} \qquad \tilde{K} = \frac{k^2}{\sqrt{2}\gamma^2} ,$$

where $\tilde{U}$ is proportional to vertical shear and $\tilde{K}$ is proportional to wavenumber, and plotting the curve of $\tilde{U}$ versus $\tilde{K}$ for the neutrally stable case of δ = 0. This plot is shown below, and demonstrates that for small values of shear the flow is stable, but as shear increases, instability will set in when $\tilde{U}$ is greater than unity. The plot also shows that there will be a certain wave number ($\tilde{K}$ = 1.0) at which the instability will first occur.



BAROCLINIC INSTABILITY IN THE ATMOSPHERE

Analysis of baroclinic instability in the real (continuously stratified) atmosphere is much more complicated than for the two-layer fluid. Qualitatively the results are similar, with instability depending on the vertical shear. However, in the real atmosphere there is always an unstable wave number, so barotropic instability is pervasive in the middle latitudes. The amount of instability, and growth rates, increase with the amount of wind shear and other factors. Baroclinic instability in the regions of strong horizontal thermal gradients is the mechanism by which midlatitude cyclones form. Baroclinic instability (combined with barotropic instability) is also important for the formation of African Easterly waves.