ESCI 343 – Atmospheric Dynamics II
Lesson 3 – QG Omega Equation

Reference: *An Introduction to Dynamic Meteorology* (3rd edition), J.R. Holton
Synoptic-dynamic Meteorology in Midlatitudes, Vol 1, H.B. Bluestein

THE QG OMEGA EQUATION

- We previously derived the QG geopotential tendency equation by eliminating omega from Eqns. (1) and (2)

$$\frac{\partial \chi}{\partial p} = -\vec{V}_g \bullet \nabla\left(\frac{\partial \Phi}{\partial p}\right) - \sigma\omega - \frac{R_d J}{p c_p}. \tag{1}$$

$$\nabla^2 \chi = -f_0 \vec{V}_g \bullet \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right) + f_0^2 \frac{\partial \omega}{\partial p}. \tag{2}$$

- To get the QG omega equation we instead eliminate the geopotential tendency from these two equations.
- This is accomplished by differentiating (2) with respect to pressure and then subtracting the horizontal Laplacian of (1).
- The result is the QG Omega Equation,

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\vec{V}_g \bullet \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)\right] - \frac{1}{\sigma}\nabla^2\left[-\vec{V}_g \bullet \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right] - \frac{R_d}{\sigma c_p}\nabla^2\left(\frac{J}{p}\right). \tag{3}$$

QG omega equation

- Like the geopotential tendency equation, we can make sense of the omega equation in a qualitative fashion my assuming that atmospheric disturbances are sinusoidal. The LHS of the equation is then proportional to the negative of omega, or

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega \propto -\omega. \tag{4}$$

- Also, we know

$$-\nabla^2\left[-\vec{V}_g \bullet \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right] \propto -\vec{V}_g \bullet \nabla\left(-\frac{\partial \Phi}{\partial p}\right)$$

$$-\nabla^2\left(\frac{J}{p}\right) \propto \frac{J}{p}$$

which allows us to write

$$\omega \propto \frac{\partial}{\partial p}\left[-\vec{V}_g \bullet \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)\right] - \left[-\vec{V}_g \bullet \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right] - \left(\frac{J}{p}\right). \quad (5)$$

or

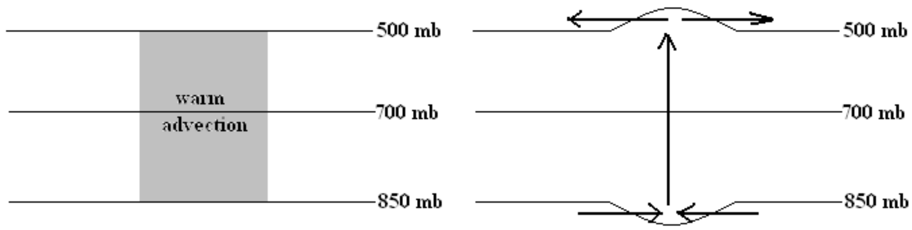
ω ∝ ∂/∂p (absolute vorticity advection) – thermal advection – heating

or

w ∝ ∂/∂z (absolute vorticity advection) + thermal advection + heating

A PHYSICAL INTERPRETATION OF THE OMEGA EQUATION

- As with the tendency equation, the terms on the RHS of the omega equation can be explained physically.
- *Differential vorticity advection*:
  - If PVA increases with height, then this implies that there will be increasing divergence of the advective wind with height.
  - This increasing divergence with height will result in upward vertical motion.
- *Thermal advection*:
  - Warm advection will increase the thickness of a layer and result in higher heights aloft compared to below.
  - This results in divergence of the isallobaric wind aloft, and convergence of the isallobaric wind below.
  - This convergence/divergence pattern leads to upward motion.



Thickness increase due to warm advection at 700 mb raises 500 mb heights and lowers 850 mb heights. The isallobaric wind diverges at 500 mb and converges at 850 mb, resulting in upward vertical motion at 700 mb.

- Note that in this case, Le Chatelier's principle is at work, because the upward motion will lead to adiabatic cooling, which opposes the temperature change forced by the advection.
- *Diabatic Heating*:
  - The diabatic heating term has essentially the same physical explanation as the advection term. Warming leads to upward motion, regardless of the cause of the warming (warm advection or diabatic heating.)

THE Q-VECTOR

- The differential vorticity advection term and the thermal advection term in the omega equation represent different physical processes.
- They may add or cancel each other, and so analysis of the net result is difficult.
- Another useful way of looking at diagnosing vertical motion is to derive an alternate form of the omega equation, called the Q-vector form of the equation.
- For the *f*-plane (constant Coriolis parameter) under adiabatic conditions the Q-vector form of the QG omega equation is

$$\sigma\nabla^2\omega + f_0^2\frac{\partial^2\omega}{\partial p^2} = -2\nabla\bullet\vec{Q}$$

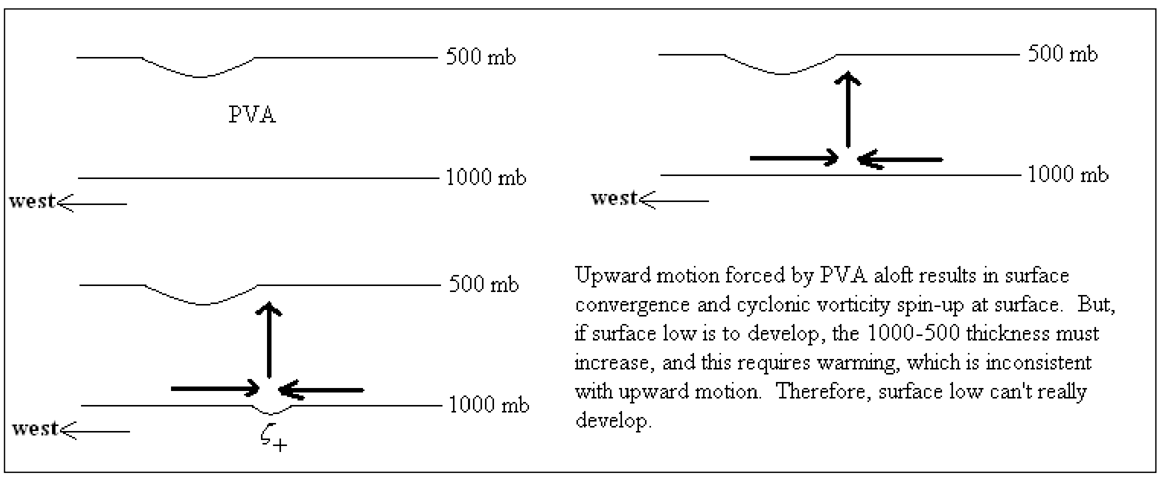
$$\vec{Q} \equiv \left(-\frac{R_d}{p}\frac{\partial\vec{V}_g}{\partial x}\bullet\nabla T, -\frac{R_d}{p}\frac{\partial\vec{V}_g}{\partial y}\bullet\nabla T\right)$$

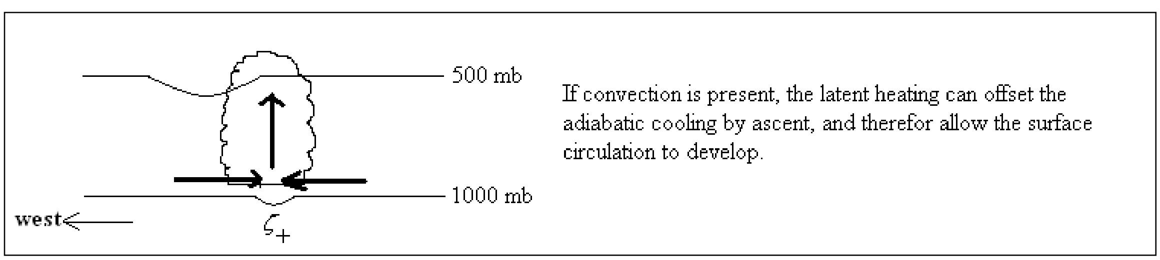
Q-vector form of omega equation

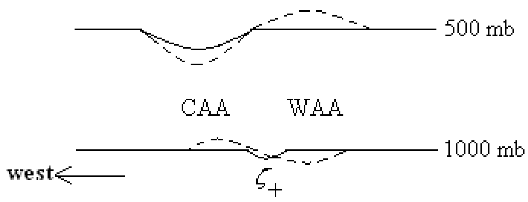
- In this form the vertical motion is only a function of the divergence of the vector **Q**. This can be analyzed on weather maps (by computers) to diagnose the omega field. The rule to remember is
  - ***DIVERGENCE OF Q MEANS DOWNWARD MOTION, CONVERGENCE OF Q MEANS UPWARD MOTION!***

THE SELF-DEVELOPMENT PROCESS FOR EXTRATROPICAL CYCLONES

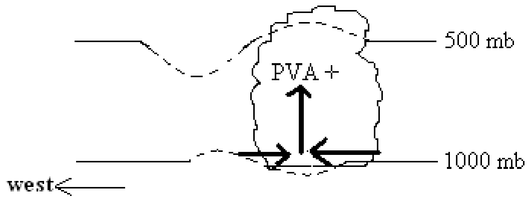
- Quasigeostrophic theory can be used to form a conceptual model of how an extratropical cyclone develops, through what is known as Pettersons's self-development process.
- This occurs when an upper-level trough approaches an old frontal boundary or baroclinic zone, and is explained below.







There will be warm-air advection in the southerly flow to the east of the surface cyclone, and cold-air advection in the northerly flow to the west of the surface cyclone. This will amplify the trough-ridge structure aloft, and also assist in propogation of the surface cyclone toward the East. The dashed lines show these forcings, but keep in mind that the upper-level low will also be propogating toward the East, so the surface low isn't going to outrun the upper-level trough.



The amplified trough-ridge structure leads to enhanced PVA over the surface low, leads to upward motion, more surface convergence, and a subsequent enhancement of the surface low. The process continues to repeat itself in a positive feedback loop.

- The diagrams above aren't meant to show exactly what the pressure surfaces will look like, but just to give an idea of how self-development operates.
- The development process will proceed as long as the upper-level trough is upstream of the surface cyclone, so that there is PVA over the surface cyclone.

EXERCISES

1. Derive the QG omega equation from the QG thermodynamic energy equation and the QG vorticity equation.