

**ESCI 343 – Atmospheric Dynamics II**  
**Lesson 1 – Ageostrophic Wind**

**References:** *An Introduction to Dynamic Meteorology* (3<sup>rd</sup> edition), J.R. Holton

**THE QG MOMENTUM EQUATIONS**

- The QG momentum equations are derived as follows:

- Start with the momentum equation in vector form,

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V} = -\nabla \Phi - \hat{k} \times f \vec{V}. \quad (1)$$

- Split the horizontal wind into geostrophic and ageostrophic components,

$$\vec{V} = \vec{V}_g + \vec{V}_a. \quad (2)$$

and substitute into (1), ignoring the ageostrophic wind in the advection term. This gives

$$\frac{\partial \vec{V}_g}{\partial t} + \vec{V}_g \bullet \nabla \vec{V}_g = -\nabla \Phi - \hat{k} \times f (\vec{V}_g + \vec{V}_a). \quad (3)$$

- Next, we replace the Coriolis parameter by  $f = f_0 + \beta y$ , so that the momentum equations become

$$\frac{\partial \vec{V}_g}{\partial t} + \vec{V}_g \bullet \nabla \vec{V}_g = -\nabla \Phi - \hat{k} \times (f_0 + \beta y) (\vec{V}_g + \vec{V}_a). \quad (4)$$

- Expand (4) to get

$$\frac{\partial \vec{V}_g}{\partial t} + \vec{V}_g \bullet \nabla \vec{V}_g = -\nabla \Phi - \hat{k} \times f_0 \vec{V}_g - \hat{k} \times f_0 \vec{V}_a - \hat{k} \times \beta y \vec{V}_g - \hat{k} \times \beta y \vec{V}_a. \quad (5)$$

- The last term in (5) is very small, and can be ignored, so we now have

$$\frac{\partial \vec{V}_g}{\partial t} + \vec{V}_g \bullet \nabla \vec{V}_g = -\nabla \Phi - \hat{k} \times f_0 \vec{V}_g - \hat{k} \times f_0 \vec{V}_a - \hat{k} \times \beta y \vec{V}_g. \quad (6)$$

- By the definition of the geostrophic wind,

$$f_0 \vec{V}_g = \hat{k} \times \nabla \Phi \quad (7)$$

so that the first two terms on the right-hand side of (6) cancel, resulting in

$$\frac{D_g \vec{V}_g}{Dt} = -\hat{k} \times f_0 \vec{V}_a - \hat{k} \times \beta y \vec{V}_g \quad (8)$$

where

$$\frac{D_g}{Dt} = \frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla. \quad (9)$$

## THE AGEOSTROPHIC WIND EQUATION

- We've been stressing the fact that though the atmosphere is close to being in geostrophic balance, the unbalanced component of the wind (the ageostrophic wind) is very important for the dynamics of synoptic disturbances.
- The ageostrophic circulation is included in QG analysis through the divergence terms.
- We are now going to derive an equation for the ageostrophic wind itself
  - We start with the QG momentum equation (8) on the  $f$ -plane, so that the beta term is zero,

$$\frac{D_g \vec{V}_g}{Dt} = -\hat{k} \times f_0 \vec{V}_a.$$

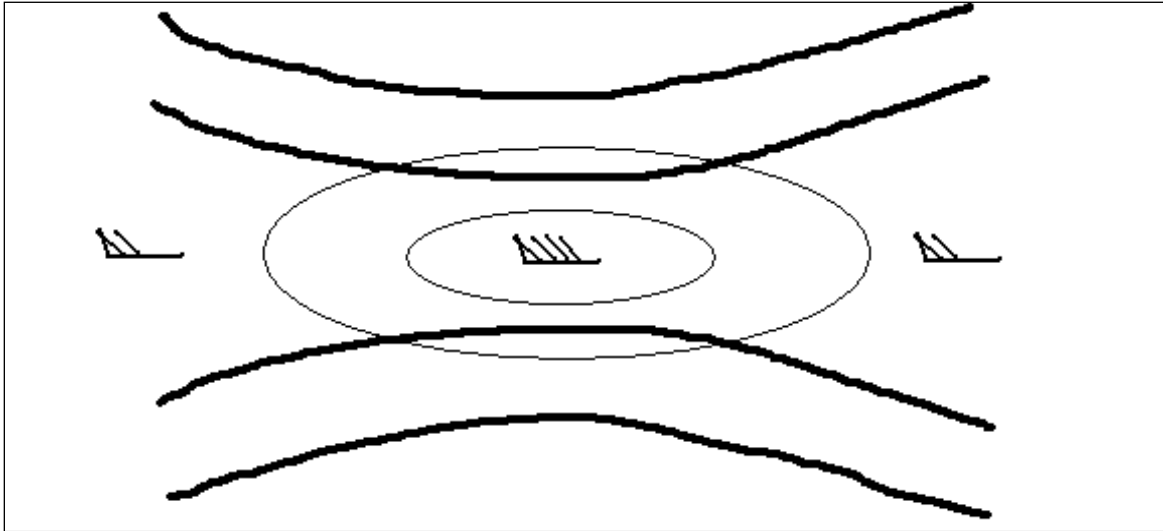
- Dividing both sides by  $f_0$  and taking  $\hat{k} \times$  of both sides results in

$$\vec{V}_a = \frac{1}{f_0} \hat{k} \times \frac{D_g \vec{V}_g}{Dt}. \quad (10)$$

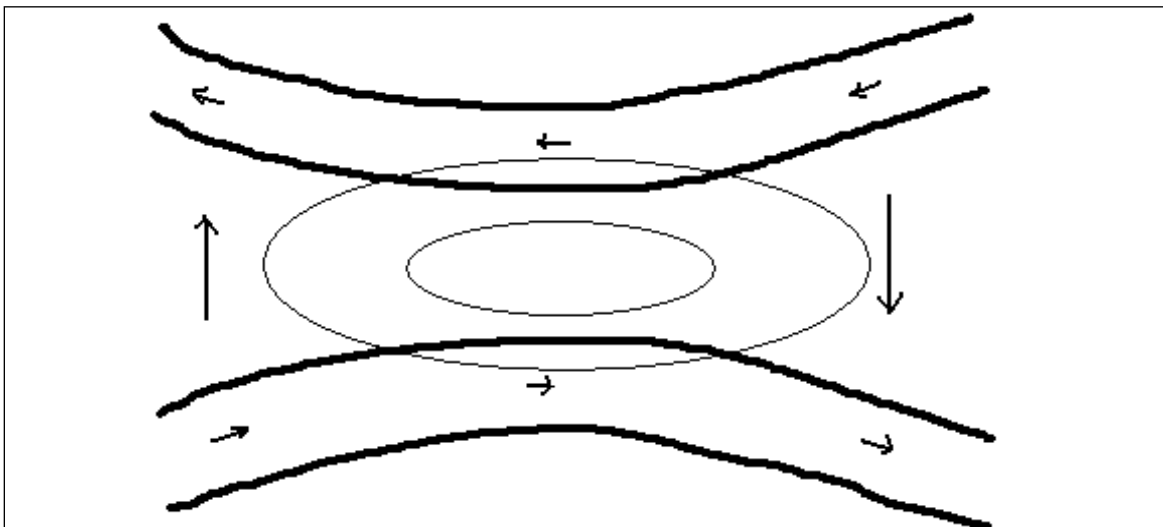
- The first thing to notice is that the ageostrophic wind always points to the left of the geostrophic acceleration (in the Northern Hemisphere).
  - This makes physical sense because if the wind flow is from an area of loose pressure gradient to tight pressure gradient, the wind must accelerate in order to get closer to geostrophic balance. The ageostrophic wind accomplishes this by blowing to the left (toward lower pressure).
  - If the flow is from tight gradient to loose gradient, the wind must decelerate, so the ageostrophic wind will blow toward higher pressure (and still left of the acceleration, which is now pointing upstream).

## AGEOSTROPHIC CIRCULATION IN JET STREAKS

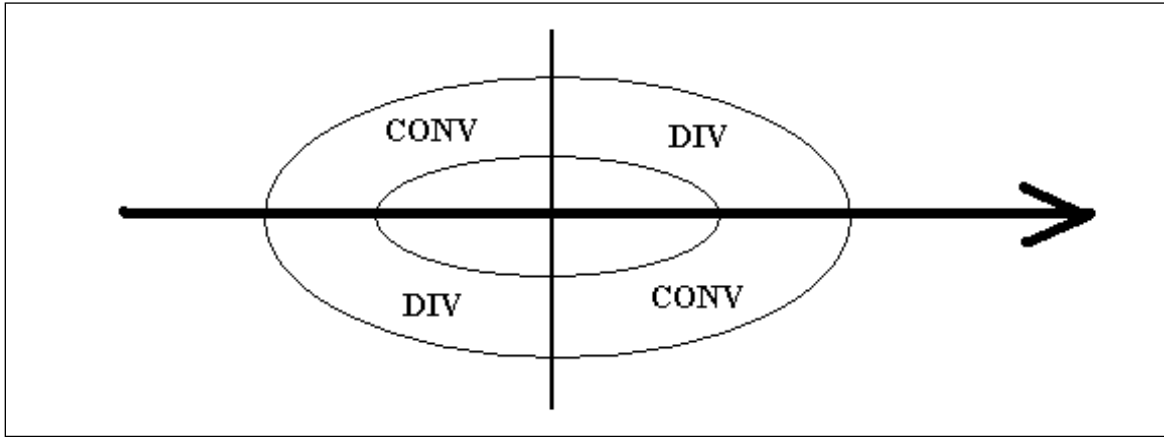
- Equation (10) can be used to explain why enhanced upward motion occurs in the right-entrance and left-exit region of jet streaks (maxima in the jet stream wind speeds).
- The diagram below shows height contours (bold) and isotachs (thin) for a typical jet streak. The wind barbs show the geostrophic wind.



- Remembering that the ageostrophic wind is to the left of the geostrophic acceleration, the ageostrophic wind will be oriented as shown below.



- From this diagram it is seen that the ageostrophic wind is divergent in the right-entrance and left-exit regions of the jet streak, and convergent in the other regions (see diagram below). This leads to upward motion in the right-entrance and left-exit regions of the jet streak.



### THE ISALLOBARIC WIND

- The total derivative in (10) can be split into a local and an advective derivative, which results in

$$\vec{V}_a = \frac{1}{f_0} \hat{k} \times \frac{\partial \vec{V}_g}{\partial t} + \frac{1}{f_0} \hat{k} \times (\vec{V}_g \cdot \nabla \vec{V}_g). \quad (11)$$

- The first term on the RHS of (11) is the *isallobaric wind*, because it can be written as

$$\vec{V}_{isall} = \frac{1}{f_0} \hat{k} \times \frac{\partial \vec{V}_g}{\partial t} = -\frac{1}{f_0^2} \nabla \left( \frac{\partial \Phi}{\partial t} \right) = -\frac{1}{f_0^2} \nabla \chi \quad (12)$$

and thus blows perpendicular to the *isallobars* (lines of constant geopotential tendency, denoted as  $\chi$ ) and toward falling heights.

- The divergence of the isallobaric wind is

$$\nabla \cdot \vec{V}_{isall} = -\frac{1}{f_0^2} \nabla^2 \chi. \quad (13)$$

- When heights are falling the isallobaric wind is convergent.
- When heights are rising the isallobaric wind is divergent.

## THE ADVECTIVE WIND

- The second term on the RHS of (11) is the *advective wind*,

$$\vec{V}_{adv} = \frac{1}{f_0} \hat{k} \times (\vec{V}_g \bullet \nabla \vec{V}_g). \quad (14)$$

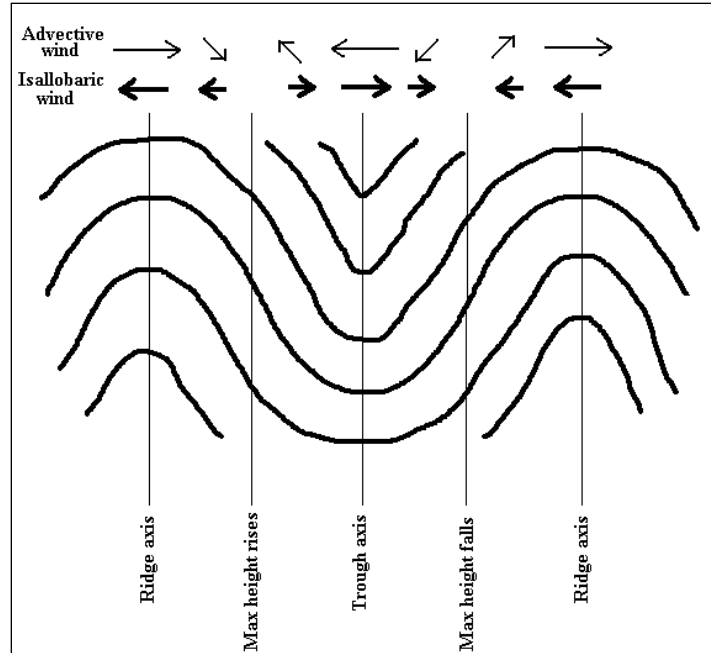
- The divergence of the advective wind is

$$\nabla \bullet \vec{V}_{adv} = -\frac{1}{f_0} \vec{V}_g \bullet \nabla \zeta_g. \quad (15)$$

- The advective wind is divergent when there is positive vorticity advection (PVA).
- The advective wind is convergent when there is negative vorticity advection (NVA).

## DIVERGENCE OF THE AGEOSTROPHIC WIND IN TROUGHS/RIDGES

- We have established that the advective wind is divergent in regions of PVA, and so
  - Downstream of a trough the advective wind is divergent
  - Upstream of trough the advective wind is convergent.
- However, if a trough is propagating in the direction of the flow then downstream of a trough the heights are falling, while upstream the heights are rising. This means that
  - Downstream of a propagating trough the isallobaric wind is convergent
  - Upstream of a propagating trough the isallobaric wind is divergent.
- The net divergence or convergence downstream of the trough depends on which is more dominant: the advective or the isallobaric wind.
  - For ***upper-level troughs*** the advective wind dominates (due to the higher wind speeds and large vorticity advection found aloft), resulting in net divergence ahead of the upper-level trough, and convergence behind it.
  - For ***lower-level troughs***, the isallobaric wind tends to either cancel or be larger than the advective wind. Therefore, there is usually net convergence ahead of a low-level trough, and net divergence behind it.
- The diagram below shows the orientation of the advective and isallobaric wind vectors for a trough-ridge system.



## EXERCISES

1. Show that  $\frac{1}{f_0} \hat{k} \times \frac{\partial \vec{V}_g}{\partial t} = -\frac{1}{f_0^2} \nabla \chi$ .

2. Show that the divergence of the isalobaric and advective winds are given by

$$\nabla \cdot \vec{V}_{iso} = -\frac{1}{f_0^2} \nabla^2 \chi$$

$$\nabla \cdot \vec{V}_{adv} = -\frac{1}{f_0} \vec{V}_g \cdot \nabla \zeta_g$$