ESCI 343 – Atmospheric Dynamics II
Answers to Selected Exercises for Lesson 12

4. Use equations 1′, 2′, 4′, and 6′ to show the following phase relations between u', v', and w' for a wave propagating in the x-z plane ($l = 0$)

d. If $w' = \cos(kx + mz - \omega t)$, what are u' and v'?

Answer: Let $\theta = kx + mz - \omega t$. Then $w' = \cos\theta$ which is the same as $\mathrm{Re}(e^{i\theta})$.

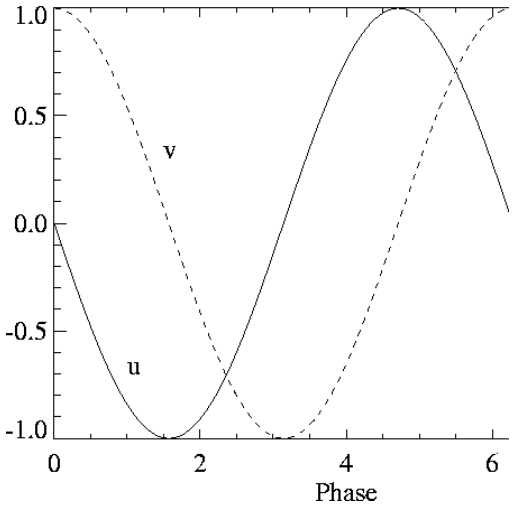
$$u' = -(m/k)w' = -(m/k)\cos\theta = -(m/k)\cos(kx + mz - \omega t)$$

$$v' = (ifm/k\omega)w' = (ifm/k\omega)\mathrm{Re}(e^{i\theta}) = \mathrm{Re}[(ifm/k\omega)e^{i\theta}] = -(fm/k\omega)\sin\theta$$

$$= -(fm/k\omega)\sin(kx + mz - \omega t)$$

5. a. Use 4.a to determine whether the horizontal velocity vector will rotate cyclonically or anticyclonically with time. Will this change if the wave is propagating upward versus downward?

Answer: From 4.a, if $v' \propto \cos\theta$ then $u' \propto -\sin\theta$. Therefore, graphs of u and v versus phase (θ) look like



which shows that as θ gets larger the velocity vector rotates counter-clockwise. Since $\theta = kx + mz - \omega t$, then as time gets bigger θ gets smaller. Therefore, the velocity vector rotates clockwise (anticyclonically) with time.

b. Use the results from 4.a to determine whether the horizontal velocity vector will rotate cyclonically or anticyclonically with height. Will this change if the wave is propagating upward versus downward?

Answer: As in part a., as θ gets larger the velocity vector rotates counter-clockwise. For an upward propagating wave θ gets larger as z increases. Therefore, the velocity vector rotates counter-clockwise (cyclonically) with height for upward propagation. For a downward propagating wave θ

gets smaller as z increases (because m <0). Therefore, the velocity vector rotates clockwise (anticyclonically) with height.

6. Show that $\vec{\mathrm{K}} \bullet \vec{V} = 0$ is the same as equation (4′).

Answer:

$$\vec{\mathrm{K}} \bullet \vec{V} = ku' + lv' + mw' = \left(kA + lB + mC\right)e^{i(kx+ly+mz-\omega t)} = 0$$

which is the same as

$$kA + lB + mC = 0\,.$$