

**ESCI 343 – Atmospheric Dynamics II**  
**Answers to Selected Exercises for Lesson 8**

3. Show that the speed of sound in an ideal gas is

$$c_s = \sqrt{\gamma R' T}.$$

**Answer:** The ideal gas law can be written (using the definition of potential temperature) as

$$p = \rho R' T = \rho R' \theta \left( \frac{p}{p_0} \right)^\kappa.$$

Differentiating with respect to density gives

$$\frac{\partial p}{\partial \rho} = R' \theta \left( \frac{p}{p_0} \right)^\kappa + \kappa \rho R' \theta \frac{p^{\kappa-1}}{p_0^\kappa} \frac{\partial p}{\partial \rho}.$$

But,

$$\theta \left( \frac{p}{p_0} \right)^\kappa = T$$

and

$$\theta \frac{p^{\kappa-1}}{p_0^\kappa} = \theta \left( \frac{p}{p_0} \right)^\kappa \frac{1}{p} = \frac{T}{p}$$

so we have

$$\frac{\partial p}{\partial \rho} = R' T + \kappa \frac{\rho R' T}{p} \frac{\partial p}{\partial \rho} = R' T + \kappa \frac{\partial p}{\partial \rho}$$

which simplifies to

$$\frac{\partial p}{\partial \rho} = \frac{1}{1 - \kappa} R' T.$$

Now, since the following are known

$$\kappa = R' / c_p$$

$$R' = c_p - c_v$$

then

$$\frac{1}{1 - \kappa} = \gamma.$$

4. a. Find the scale height and speed of sound for an isothermal atmosphere with a temperature of 255K.

**Answer:**  $H = 7463$  m;  $c_s = 320$  m/s

- b. For this atmosphere, find out how large the wavelength of an acoustic wave would need to be before we started concerning ourselves with the effects of gravity and buoyancy on these waves.

**Answer:**

$$k = \frac{2\pi}{\lambda} \gg -\frac{1}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial z}, \text{ which requires } \lambda \ll 2\pi \left( -\frac{1}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial z} \right)^{-1}.$$

For an isothermal atmosphere,

$$\bar{\rho} = \rho_0 \exp(-z/H)$$

and

$$\frac{1}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial z} = -\frac{1}{H}.$$

Therefore,

$$\lambda \ll 2\pi H = 47 \text{ km}$$

Therefore, as long as our sound waves have wavelengths less than about 5 km we can ignore the effects of gravity.