ESCI 343 – Atmospheric Dynamics II
Selected Answers for Exercises for Lesson 6

1. Find the dispersion relation for one-dimensional (*x*-direction) shallow-water gravity waves with a non-zero mean flow in the zonal direction (i.e., $\bar{u} \neq 0$, $\bar{v} = 0$).

Answer: $\omega = \bar{u}k \pm k\sqrt{gH}$ **.** Note, this is for propagation in the *x*-direction only.

2. a. Find the dispersion relation for two-dimensional (*x*- and *y*-directions) shallow-water gravity waves with a zero mean flow, but including the Coriolis parameter (these are known as *shallow-water, intertio-gravity waves*).

Answer: $\omega^2 = gH\left(k^2 + l^2\right) + f^2 = gH\mathrm{K}^2 + f^2$

b. Find the group velocity and phase speed of these waves. Are they dispersive?

Answer:

$$c = \sqrt{gH + f^2/\mathrm{K}^2}$$

$$c_g = \frac{gH\mathrm{K}}{\sqrt{gH\mathrm{K}^2 + f^2}} = \frac{gH}{\sqrt{gH + f^2/\mathrm{K}^2}}$$

Yes, they are dispersive.

3. The general dispersion relation for one-dimensional surface gravity waves (not restricted to shallow water) is

$$\omega = \bar{u}k \pm \sqrt{gk \tanh kH}\ .$$

a. What is the phase speed for these waves?

Answer: $c = \bar{u} \pm \sqrt{(g/k)\tanh kH}$

b. Are these waves dispersive?

Answer: Yes

6. Calculate the speed of a surface gravity wave for a fluid having a depth equal to the scale height of the atmosphere (~8.1 km). How does this compare with the speed of sound?

Answer: 282 m/s (the speed of sound is ~ 300 m/s)