

**ESCI 342 – Atmospheric Dynamics I**  
**Lesson 8 – Geostrophic and Gradient Balance**

**Suggested Reading:** Martin, Chapter 4 (except Section 4.3)

**PRESSURE GRADIENT ACCELERATION IN HEIGHT AND PRESSURE COORDINATES**

- In a prior lesson we've already established that the pressure gradient acceleration, which in height ( $z$ ) coordinates is

$$PGA = -\frac{1}{\rho} \nabla p, \quad (1)$$

is in pressure coordinates given as

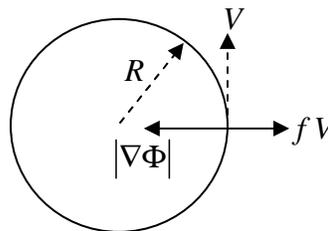
$$PGA = -\nabla\Phi, \quad (2)$$

where  $\Phi$  is geopotential.

- Since (2) is simpler and easier to write, we will use pressure coordinates in this lesson.
- Keep in mind that the equations we develop in this lesson can be converted to height coordinates simply by replacing  $\nabla\Phi$  or spatial derivatives of  $\Phi$   $\nabla p/\rho$  or its spatial derivatives.

**GRADIENT WIND AROUND A LOW PRESSURE**

- The diagram below shows the directions of the Coriolis and pressure-gradient accelerations for normal flow around a low. The radius of the flow is  $R$ , and the speed is  $V$ .



- The balance of acceleration in the radial direction is

$$\frac{V^2}{R} = |\nabla\Phi| - fV, \quad (3)$$

which can be rearranged to

$$V^2 + fRV - R|\nabla\Phi| = 0. \quad (4)$$

- Equation (4) is quadratic in  $V$ , and using the quadratic formula is solved for  $V$  as

$$V = -\frac{fR}{2} \pm \frac{1}{2} \sqrt{f^2 R^2 + 4R|\nabla\Phi|}. \quad (5)$$

- Equation (5) gives the gradient wind speed around a low pressure. What is interesting is that there are two solutions, corresponding to the two roots of the radical.
  - The first solution, with the + sign, is

$$V = -\frac{fR}{2} + \frac{1}{2}\sqrt{f^2R^2 + 4R|\nabla\Phi|} , \quad (6)$$

and is the gradient wind around a *regular low*.

- o The second solution, with the  $-$  sign, is

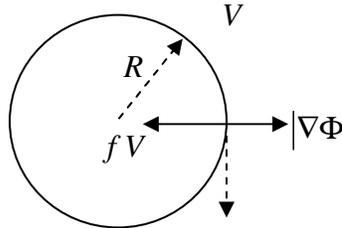
$$V = -\frac{fR}{2} - \frac{1}{2}\sqrt{f^2R^2 + 4R|\nabla\Phi|} , \quad (7)$$

and is the gradient wind around an *anomalous low*.

- o The winds for the anomalous low are very large, and actually in the opposite direction of the regular low (since the speed is negative).
- o In the anomalous low, the pressure-gradient and Coriolis accelerations both point toward the center and contribute to the centripetal acceleration.

### GRADIENT WIND AROUND A HIGH PRESSURE

- The diagram below shows the directions of the Coriolis and pressure-gradient accelerations for normal flow around a high.



- In order to be balanced, the pressure gradient acceleration and the Coriolis must sum up to equal the centripetal accelerations, which is  $V^2/R$ . Therefore, the following equation must be true,

$$V^2/R = fV - |\nabla\Phi| ,$$

which can be rearranged to

$$V^2 - fRV + R|\nabla\Phi| = 0 . \quad (8)$$

- Solving (8) for  $V$  yields

$$V = \frac{fR}{2} \pm \frac{1}{2}\sqrt{f^2R^2 - 4R|\nabla\Phi|} . \quad (9)$$

- Equation (9) gives the gradient wind speed around a high pressure. As with the low, there are two solutions, corresponding to the two roots of the radical.

- o The first solution, with the  $+$  sign, is

$$V = \frac{fR}{2} + \frac{1}{2}\sqrt{f^2R^2 - 4R|\nabla\Phi|} , \quad (10)$$

and is the gradient wind around an *anomalous high*.

- o The second solution, with the  $-$  sign, is

$$V = \frac{fR}{2} - \frac{1}{2}\sqrt{f^2R^2 - 4R|\nabla\Phi|} , \quad (11)$$

and is the gradient wind around a *regular high*.

- o The winds for both the anomalous and regular high rotate in the normal sense, but the speed for the anomalous high is very large.

- For highs, there is a restriction on how strong the pressure gradient may be.
  - Notice in (10) and (11) that in order for the wind speed to be “real” and not have an “imaginary” component, the following condition must hold

$$f^2 R^2 - 4R|\nabla\Phi| \geq 0. \quad (12)$$

- This requires that

$$|\nabla\Phi| \leq \frac{f^2}{4}|R|. \quad (13)$$

- This explains why, on a synoptic scale weather map, we often see tightly wound lows with large pressure gradients right down to the center, while we never see large pressure gradients in the center of high pressures. *For the high pressure case, there is a physical limit as to how large the pressure gradient can be near the center.*

### MORE ON GRADIENT WIND

- Under most large-scale flow regimes the atmosphere is close to being in gradient balance, and the gradient wind equations, (6) and (11), are appropriate to use for the low and high respectively.
- The anomalous low and high are not readily observed in the atmosphere, primarily because anomalous low and high both have negative absolute vorticity, and it is difficult to imagine a situation where a large scale circulation can develop having negative absolute vorticity.
- It may be possible to observe anomalous gradient flows on the small scale. However, such flows are actually likely to have large Rossby numbers, and be closer to cyclostrophic balance rather than in gradient balance.
- For more information on the anomalous solutions refer to:
  - Fultz, 1991: “Quantitative nondimensional properties of the gradient wind”, *J. Atmos. Sci.*, **48**, 869-875
  - Chew, F. and M.H. Bushnesll, 1990, “The half-inertial flow in the Eastern Equatorial Pacific: A case study”, *J. Phys. Ocean.*, **20**, 1124-1133
  - Mogil, H.M. and R.L. Holle, 1972: “Anomalous gradient winds: Existence and implications”, *Mon. Wea. Rev.*, **100**, 709-716
  - Alaka, M.A., 1961: “The occurrence of anomalous winds and their significance”, *Mon. Wea. Rev.*, **89**, 482-494
- Note that the anomalous high exhibits the curious behavior that the wind speed actually increases as the pressure gradient force decreases!
- The anomalous high and anomalous low both become inertial flow as the pressure gradient goes to zero.

### SIMPLIFIED EXPRESSION OF THE GRADIENT WIND EQUATION

- The gradient wind equation can also be written in terms of the geostrophic wind speed. Since, by definition,

$$V_g = |\nabla\Phi|/f, \quad (14)$$

the gradient wind for the regular cyclone and anticyclone can be expressed as

$$\text{Cyclone:} \quad V_{gr} = \frac{fR}{2} \left( \sqrt{1 + \frac{4V_g}{fR}} - 1 \right) \quad (15)$$

$$\text{Anticyclone:} \quad V_{gr} = \frac{fR}{2} \left( 1 - \sqrt{1 - \frac{4V_g}{fR}} \right) \quad (16)$$

### TRAJECTORIES VS. STREAMLINES

- A *trajectory* is a curve tracing the successive points of the particles position in time.
- A *streamline* is a line that is tangent to the velocity at a point, at a given instant.
- Trajectories and streamlines only coincide if the fluid motion is steady.
- The local rate of change of wind direction is

$$\frac{\partial \beta}{\partial t} = V \left( \frac{1}{R_T} - \frac{1}{R_S} \right)$$

where  $R_T$  and  $R_S$  are the radii of curvature for the trajectory and the streamline respectively.

### EXERCISES

1. For the same values of pressure gradient, Coriolis parameter, and radius of curvature, the flow around a regular low is slower than the flow around a regular high. Give a physical explanation of this fact. Drawing force diagrams should be very helpful.
2. The geostrophic wind speed is given as

$$V_g = \frac{|\nabla \Phi|}{f}.$$

Use this to derive (15) and (16).