

ESCI 342 – Atmospheric Dynamics I
Answers to Selected Exercises for Lesson 13

5. Show that $\frac{\partial}{\partial x} \frac{D}{Dt} \neq \frac{D}{Dt} \frac{\partial}{\partial x}$ (i.e., material derivatives don't commute with partial derivatives).

Answer:

$$\begin{aligned} \frac{\partial}{\partial x} \frac{D}{Dt} &= \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} + \vec{V} \cdot \nabla \right) = \frac{\partial}{\partial x} \frac{\partial}{\partial t} + \frac{\partial}{\partial x} (\vec{V} \cdot \nabla) = \frac{\partial}{\partial x} \frac{\partial}{\partial t} + \vec{V} \cdot \frac{\partial}{\partial x} \nabla + \frac{\partial \vec{V}}{\partial x} \cdot \nabla \\ &= \frac{\partial}{\partial t} \frac{\partial}{\partial x} + \vec{V} \cdot \nabla \frac{\partial}{\partial x} + \frac{\partial \vec{V}}{\partial x} \cdot \nabla = \frac{D}{Dt} \frac{\partial}{\partial x} + \frac{\partial \vec{V}}{\partial x} \cdot \nabla \end{aligned}$$

so

$$\frac{\partial}{\partial x} \frac{D}{Dt} = \frac{D}{Dt} \frac{\partial}{\partial x} + \frac{\partial \vec{V}}{\partial x} \cdot \nabla$$