ESCI 342 – Atmospheric Dynamics I
Answers to Selected Exercises for Lesson 10

2. Show that

$$\frac{\partial T}{\partial p} - \frac{\alpha}{c_p} = \left(\frac{\alpha}{\theta} \frac{\partial \theta}{\partial p} \right) \frac{p}{R_d} .$$

Hint: Take $\partial / \partial p$ of $T = \theta (p / p_0)^{\kappa}$.

Answer:

$$\frac{\partial T}{\partial p} = \kappa \theta \frac{p^{\kappa - 1}}{p_0^{\kappa}} + \left(\frac{p}{p_0} \right)^{\kappa} \frac{\partial \theta}{\partial p} = \frac{\kappa}{p} \theta \left(\frac{p}{p_0} \right)^{\kappa} + \left(\frac{p}{p_0} \right)^{\kappa} \frac{\partial \theta}{\partial p} =$$

$$\kappa \frac{T}{p} + \frac{T}{\theta} \frac{\partial \theta}{\partial p} = \frac{R_d}{c_p} \frac{T}{p} + \frac{p}{R_d} \frac{\alpha}{\theta} \frac{\partial \theta}{\partial p} = \frac{\alpha}{c_p} + \frac{p}{R_d} \frac{\alpha}{\theta} \frac{\partial \theta}{\partial p}$$

so

$$\frac{\partial T}{\partial p} - \frac{\alpha}{c_p} = \frac{p}{R_d} \frac{\alpha}{\theta} \frac{\partial \theta}{\partial p}$$

4. Use the adiabatic method to estimate the 500 mb vertical velocity (ω) for the following situation. The temperature tendency is zero. The temperature at 600 mb is −13°C, at 500 mb it is −19°C. The wind at 500 mb is from the SW at 20 m/s, and the temperature at 500 mb increases toward the West at 1°C/100 km.

Answer:

$$\omega = \frac{R_d}{\sigma p} \vec{V} \bullet \nabla_p T$$

$\theta_{600} = 300.9$ K, $\theta_{500} = 309.7$ K so $\dfrac{\partial \theta}{\partial p} \cong \dfrac{300.9 K - 309.7 K}{10000 Pa} = -8.8 \times 10^{-4} \; K/Pa$

$$\alpha = \frac{R_d T}{p} = 1.46 m^3/kg \quad \text{so} \quad \sigma = -\frac{\alpha}{\theta} \frac{\partial \theta}{\partial p} = 4.15 \times 10^{-6} \; m^3/(kg \; Pa)$$

$$\vec{V} \bullet \nabla_p T = (20 m/s)(10^{-5} \; °C/m) \cos 135° = -1.41 \times 10^{-4} \; °C/s$$

$$\omega = -0.196 \, Pa/s = -7.06 \, mb/hr$$