ESCI 342 – Atmospheric Dynamics
Answers to Selected Exercises for Lesson 7

2. Show that $\lim_{\delta x,\delta y,\delta z\to 0} \frac{1}{\delta x\delta y\delta z}\frac{D}{Dt}(\delta x\delta y\delta z) = \nabla\bullet\vec{V}$. Hint: You will need to convince yourself that $\frac{D}{Dt}(\delta x) = \delta u;\quad \frac{D}{Dt}(\delta y) = \delta v;\quad \frac{D}{Dt}(\delta z) = \delta w$.

Answer: $\frac{D}{Dt}(\delta x\delta y\delta z) = \delta y\delta z\frac{D(\delta x)}{Dt} + \delta x\delta z\frac{D(\delta y)}{Dt} + \delta x\delta y\frac{D(\delta z)}{Dt}$

but

$$\frac{D}{Dt}(\delta x) = \frac{D}{Dt}(x_2 - x_1) = u_2 - u_1 = \delta u$$

and similarly

$$\frac{D}{Dt}(\delta y) = \delta v;\quad \frac{D}{Dt}(\delta z) = \delta w$$

so we have

$$\frac{D}{Dt}(\delta x\delta y\delta z) = \delta y\delta z\delta u + \delta x\delta z\delta v + \delta x\delta y\delta w$$

and therefore

$$\frac{1}{\delta x\delta y\delta z}\frac{D}{Dt}(\delta x\delta y\delta z) = \frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z}$$

which in the limit as δx, δy, and δz approach zero becomes $\nabla\bullet\vec{V}$.