**ESCI 342 – Atmospheric Dynamics**
**Selected Answers to Exercises for Lesson 6**

**1.** Perform a scale analysis of the horizontal momentum equations (in component form) for the whirlpool formed as your bathroom sink drains. Which terms are important in this case? Water has a density of 1000 kg/m$^3$ and a kinematic viscosity of $\nu$ = 1.8×10$^{-6}$ m$^2$ s$^{-1}$. The horizontal pressure difference across the whirlpool is ~ 10 Pa. (Use a reasonable estimate for the horizontal velocity based on your own experiences.)

**Answer:**

| **Name** | **Symbol** | **Order of magnitude** |
|---|---|---|
| Horizontal velocity | $U$ | 0.01 m/s |
| Vertical velocity | $W$ | 0.01 m/s |
| Horizontal distance | $L$ | 0.01 m |
| Vertical distance | $H$ | 0.1 m |
| Horizontal pressure change | $\delta P$ | 0.1 Pa |
| Density | $\rho$ | 1000 kg/m$^3$ |
| Time | $\tau = L/U$ | 10 s |
| Kinematic viscosity | $\nu$ | 1.8×10$^{-6}$ m$^2$/s |
| Omega | $\Omega$ | 7.292×10$^{-5}$ rad/s |
| Latitude | $\phi$ | 45° |
| Radius of Earth | $a$ | 6.378×10$^6$ m |

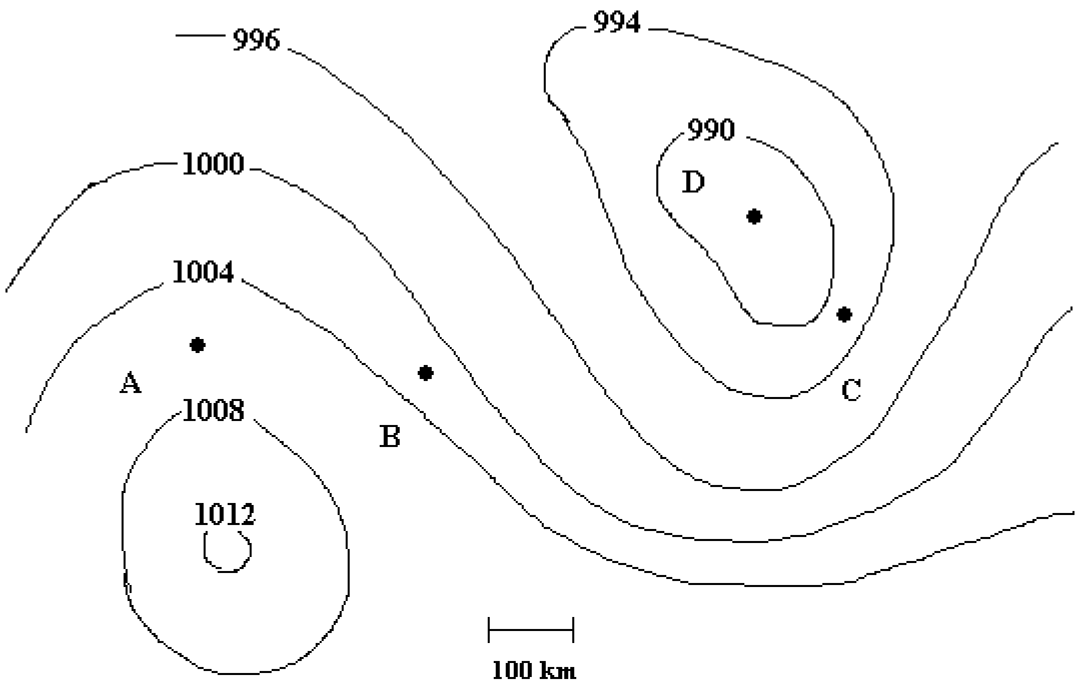
| $\frac{\partial u}{\partial t}$ | $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}$ | $w\frac{\partial u}{\partial z}$ | $-\frac{uv\tan\phi}{a}$ | $\frac{uw}{a}$ | $-\frac{1}{\rho}\frac{\partial p}{\partial x}$ | $2\Omega v\sin\phi$ | $-2\Omega w\cos\phi$ | $\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$ | $\nu\frac{\partial^2 u}{\partial z^2}$ |
|---|---|---|---|---|---|---|---|---|---|
| $U^2/L$ | $U^2/L$ | $WU/H$ | $U^2/a$ | $UW/a$ | $\delta P/(\rho L)$ | $2\Omega U \sin 45$ | $2\Omega W \cos 45$ | $\nu U/L^2$ | $\nu U/H^2$ |
| $10^{-2}$ m/s$^2$ | $10^{-2}$ m/s$^2$ | $10^{-3}$ m/s$^2$ | $10^{-11}$ m/s$^2$ | $10^{-11}$ m/s$^2$ | $10^{-2}$ m/s$^2$ | $10^{-6}$ m/s$^2$ | $10^{-6}$ m/s$^2$ | $10^{-4}$ m/s$^2$ | $10^{-6}$ m/s$^2$ |

| $\frac{\partial v}{\partial t}$ | $u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}$ | $w\frac{\partial v}{\partial z}$ | $\frac{u^2\tan\phi}{a}$ | $\frac{vw}{a}$ | $-\frac{1}{\rho}\frac{\partial p}{\partial y}$ | $-2\Omega u\sin\phi$ | $\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$ | $\nu\frac{\partial^2 v}{\partial z^2}$ |
|---|---|---|---|---|---|---|---|---|
| $U^2/L$ | $U^2/L$ | $WU/H$ | $UV/a$ | $UW/a$ | $\delta P/(\rho L)$ | $2\Omega U \sin 45$ | $\nu U/L^2$ | $\nu U/H^2$ |
| $10^{-2}$ m/s$^2$ | $10^{-2}$ m/s$^2$ | $10^{-3}$ m/s$^2$ | $10^{-11}$ m/s$^2$ | $10^{-11}$ m/s$^2$ | $10^{-2}$ m/s$^2$ | $10^{-6}$ m/s$^2$ | $10^{-4}$ m/s$^2$ | $10^{-6}$ m/s$^2$ |

Pressure gradient and acceleration are dominant terms. Flow is cyclostrophic.

**2.** What is the Rossby number for a tornado? Does the Coriolis force effect a tornado? **Answer:** Using $U$ ~ 100 m/s and $L$ ~ 100 m, $Ro$ ~ 10$^4$. Therefore, Coriolis does not influence tornadoes.

**5.** At the four points shown in the picture below, estimate the magnitude of the geostrophic wind. Assume a density of 1.23 kg/m$^3$ and a latitude of 45°. The isobars are labeled in mb.




**Answer:** **A** - 19 m/s; **B** - 39 m/s; **C** - 58 m/s; **D** – 0 (answers will vary slightly due to measurement error).

**6.** Perform a scale analysis of the vertical momentum equation for a thunderstorm to find out what terms can be ignored.

**Answer:** Using $U \sim W \sim 10$ m/s and $L \sim H \sim 10^4$ m, the resulting equation is

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g.$$