ESCI 342 – Atmospheric Dynamics I
Selected to Answers Exercises for Lesson 5

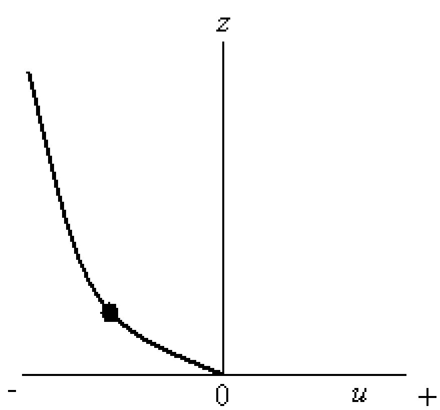
EXERCISES

1. Show that if the wind is blowing parallel to the isotherms that the temperature advection is zero.

Answer: If wind is parallel to isobars, then ***V*** and ∇T are normal to each other. Therefore, their dot product is zero.

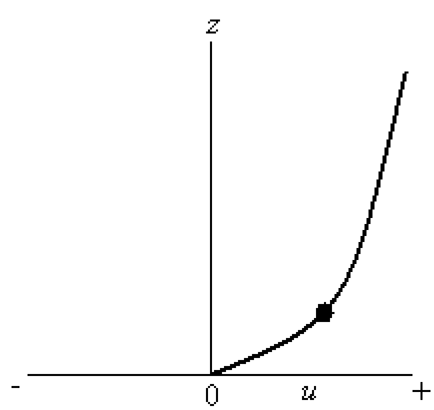
6. a. For the following profile of *u*, explain whether a downdraft would cause an increase or decrease in *u* at the location of the dot. Assume that *u* is constant in *x* and *y* [$u = u(t, z)$]

Hint: $\frac{\partial u}{\partial t} = -\vec{V} \bullet \nabla u = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z}$



Answer: $\frac{\partial u}{\partial t} = -w\frac{\partial u}{\partial z}$; $w < 0, \frac{\partial u}{\partial z} < 0 \quad \therefore \frac{\partial u}{\partial t} < 0$

b. Do the same as in 6.a., only for the following profile



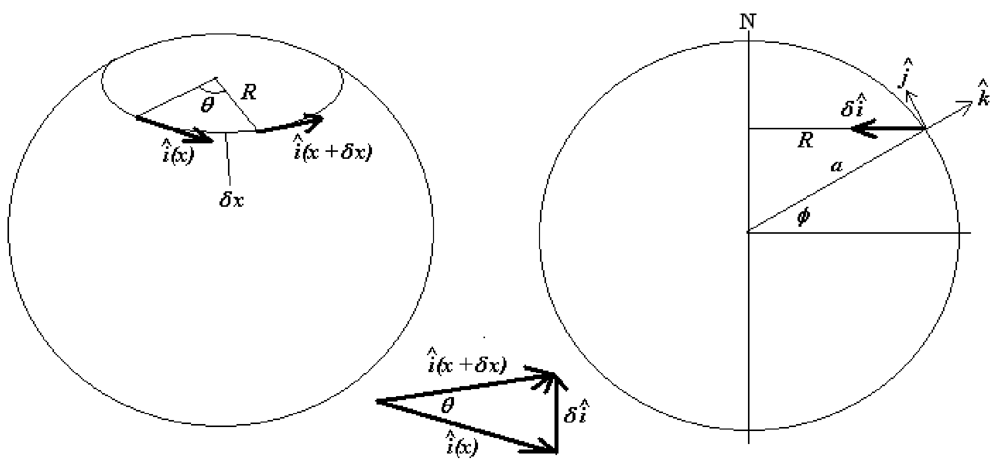
Answer: $\frac{\partial u}{\partial t} = -w\frac{\partial u}{\partial z}$; $w < 0, \frac{\partial u}{\partial z} > 0 \quad \therefore \frac{\partial u}{\partial t} > 0$

7. Prove the following identities:

$\dfrac{\partial \hat{i}}{\partial x} = \dfrac{\tan\phi}{a}\hat{j} - \dfrac{1}{a}\hat{k}$	$\dfrac{\partial \hat{j}}{\partial x} = -\dfrac{\tan\phi}{a}\hat{i}$	$\dfrac{\partial \hat{k}}{\partial x} = \dfrac{1}{a}\hat{i}$
$\dfrac{\partial \hat{i}}{\partial y} = 0$	$\dfrac{\partial \hat{j}}{\partial y} = -\dfrac{1}{a}\hat{k}$	$\dfrac{\partial \hat{k}}{\partial y} = \dfrac{1}{a}\hat{j}$
$\dfrac{\partial \hat{i}}{\partial z} = 0$	$\dfrac{\partial \hat{j}}{\partial z} = 0$	$\dfrac{\partial \hat{k}}{\partial z} = 0$

Answer for $\partial\hat{i}/\partial x$ and $\partial\hat{j}/\partial x$, which are the most difficult.

$\dfrac{\partial \hat{i}}{\partial x}$**:** Refer to diagram below:



From the diagrams above we see that

$$\left|\frac{\partial \hat{i}}{\partial x}\right| = \lim_{\delta x\to 0}\frac{\left|\delta\hat{i}\right|}{\delta x} = \lim_{\delta x\to 0}\frac{\left|\hat{i}\left(x+\delta x\right)-\hat{i}\left(x\right)\right|}{\delta x} = \lim_{\delta x\to 0}\frac{\theta}{\delta x}.$$

The angle θ is also equal to

$$\theta = \frac{\delta x}{R} = \frac{\delta x}{a\cos\phi}$$

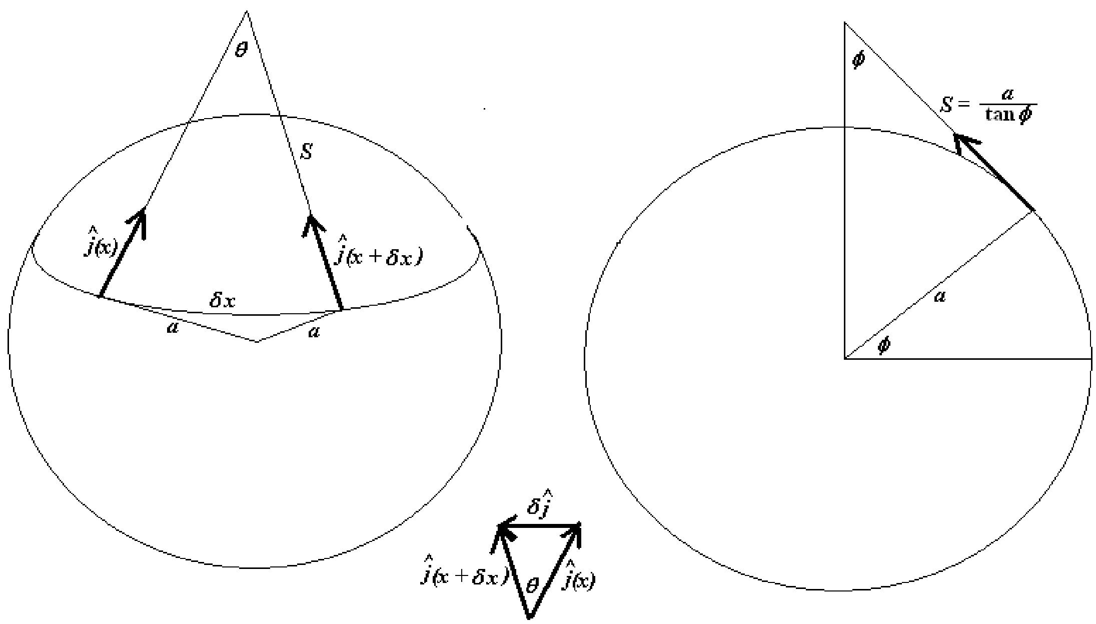
so that

$$\left|\frac{\partial \hat{i}}{\partial x}\right| = \lim_{\delta x\to 0}\frac{\theta}{\delta x} = \frac{1}{a\cos\phi}.$$

The direction is oriented along the line perpendicular to the axis of rotation, so from the diagram on the left we see that

$$\frac{\partial \hat{i}}{\partial x} = \left|\frac{\partial \hat{i}}{\partial x}\right|\left(\sin\phi\,\hat{j} - \cos\phi\,\hat{k}\right) = \frac{1}{a\cos\phi}\left(\sin\phi\,\hat{j} - \cos\phi\,\hat{k}\right) = \frac{\tan\phi}{a}\hat{j} - \frac{1}{a}\hat{k}$$

$\frac{\partial \hat{j}}{\partial x}$**:** Refer to diagram below:



From the diagrams above we see that

$$\left|\frac{\partial \hat{j}}{\partial x}\right| = \lim_{\delta x \to 0} \frac{\left|\delta \hat{j}\right|}{\delta x} = \lim_{\delta x \to 0} \frac{\left|\hat{j}\left(x+\delta x\right) - \hat{j}(x)\right|}{\delta x} = \lim_{\delta x \to 0} \frac{\theta}{\delta x}.$$

The angle θ is also equal to

$$\theta = \frac{\delta x}{S} = \frac{\delta x}{a/\tan\phi}$$

so that

$$\left|\frac{\partial \hat{j}}{\partial x}\right| = \lim_{\delta x \to 0} \frac{\theta}{\delta x} = \frac{\tan\phi}{a}.$$

The direction is oriented westward, so that

$$\frac{\partial \hat{j}}{\partial x} = -\frac{\tan\phi}{a}\hat{i}$$