ESCI 342 – Atmospheric Dynamics I
Selected Answers to Exercises for Lesson 3

3. b. Using the results from part a., find the magnitude of the gravity force at the North Pole, 45°N, and at the Equator ($g^* = 9.81$ m/s^2, the radius of the Earth is 6378 km, and $\Omega = 7.292\times10^{-5}$ rad/s).

Answer: North Pole: 9.81 m/s^2
45°N: 9.79 m/s^2
Equator: 9.78 m/s^2

c. Using your results for the gravity force from part a., find the geopotential height at an altitude of 5000 meters at the North Pole, 45°N, and at the Equator (assume that the gravity force is constant with height).

Answer: North Pole: 5001.7 m
45°N: 4991.5 m
Equator: 4986.4 m

4. Assume that the gravity force at the surface is $g_0 = 9.80665$ m/s^2. Calculate the geopotential height at an altitude of 5,000 meters for the following two cases:

a. Gravity is constant with height.

Answer: 5000 m

b. Gravity decreases with height according to the following formula,

$$g = \frac{g_0}{(1 + z/a)^2}$$

where a is the radius of the Earth (6378 km).

Answer: 4996.1 m

6. Holton problem 1.1

Answer: From the diagram below we see that

$$\sin\alpha = g_y/g$$

From Problem 3 we know that

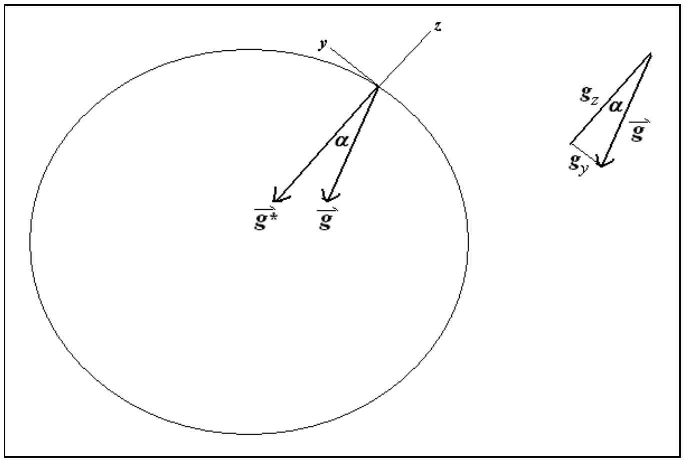
$$g_y = \Omega^2 a\cos\phi\sin\phi$$

so that

$$\sin\alpha = \frac{\Omega^2 a\cos\phi\sin\phi}{g} = \frac{\Omega^2 a\sin 2\phi}{2g}.$$

For very small angles the sine is equal to the angle itself (in radians), so we get that

$$\alpha \cong \frac{\Omega^2 a \sin 2\phi}{2g}$$



9. Holton problem 1.7

Answer: Lateral force is 995 N. Traveling east, vertical force is 1066 N. Traveling west, vertical force is –1066 N.

10. An ant is walking on a turntable that is rotating clockwise at 5 revolutions per minute (rpm). A coordinate system (x', y')is rotating with the turntable, the origin of which is the center of the turntable, with the y'-axis points radially outward. At time $t = 0$, this coordinate system is perfectly aligned with a coordinate system fixed to the non-rotating room (x, y). The ant is initially at coordinates $x = x' = 0$, $y = y' = 1$ cm, and with respect to the turntable is traveling along the y'-axis at a constant speed of 0.5 cm/s.

a. What is the angular velocity of the turntable in rad/s?

Answer: -0.52 rad s^{-1} $\hat{k}$

b. What are the components (in the rotating reference frame) of the ant's Coriolis acceleration at time $t = 0$?

Answer: -5.2×10^{-3} m s^{-2} $\hat{\imath}$

c. What are the components (in the rotating reference frame) of the ant's centrifugal acceleration at time $t = 0$?

Answer: 2.7×10^{-3} m s^{-2} $\hat{\jmath}$

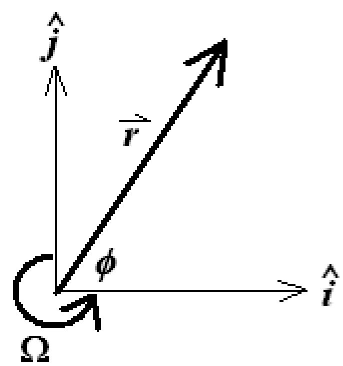
d. What are the components of the ant's velocity in the reference frame fixed to the room?

Answer: $5.2\times10^{-3}\ \text{m s}^{-1}\ \hat{i} + 5.0\times10^{-3}\ \text{m s}^{-1}\ \hat{j}$

e. What are the components of the ant's acceleration in the reference frame fixed to the room?

Answer: $5.2\times10^{-3}\ \text{m s}^{-2}\ \hat{i} - 2.7\times10^{-3}\ \text{m s}^{-2}\ \hat{j}$

11. A coordinate system is rotating counter-clockwise around it's $\hat{k}$ axis at an angular speed Ω. An object is located at position $\vec{r}$, and has a velocity such that the Coriolis acceleration is exactly balanced by the centrifugal acceleration.



a. What are the *u* and *v* components of the velocity of the object in the rotating coordinate system?

Answer: $\vec{V}_r = \frac{\Omega r}{2}\left(\sin\phi\ \hat{i} - \cos\phi\ \hat{j}\right)$

b. What are the *u* and *v* components of the velocity of the object in a non-rotating coordinate system?

Answer: $\vec{V}_a = \frac{\Omega r}{2}\left(-\sin\phi\ \hat{i} + \cos\phi\ \hat{j}\right)$

c. What will the path of the object look like in the rotating coordinate system?

Answer: Clockwise circle of radius *r*.

d. What will the path of the object look like in a non-rotating coordinate system?

Answer: Counter-clockwise circle of radius *r*.

12. Show that $\nabla^2 \vec{V}_a = \nabla^2 \vec{V}_r$. Hint: To do this, you will need to show that $\nabla^2 \left(\vec{\Omega} \times \vec{r} \right) = 0$. Use the identity $\nabla^2 \vec{A} = \nabla \left(\nabla \bullet \vec{A} \right) - \nabla \times \left(\nabla \times \vec{A} \right)$, and recognize that $\vec{\Omega} \times \vec{r}$ is the tangential velocity of solid-body rotation. Also, the velocity divergence in solid-body rotation is zero, and the vorticity in solid-body rotation is constant.

Answer: $\vec{V}_a = \vec{V}_r + \vec{\Omega} \times \vec{r}$ so $\nabla^2 \vec{V}_a = \nabla^2 \left(\vec{V}_r + \vec{\Omega} \times \vec{r} \right) = \nabla^2 \vec{V}_r + \nabla^2 \left(\vec{\Omega} \times \vec{r} \right)$.

$\nabla^2 \left(\vec{\Omega} \times \vec{r} \right) = \nabla \left[\nabla \bullet \left(\vec{\Omega} \times \vec{r} \right) \right] - \nabla \times \left[\nabla \times \left(\vec{\Omega} \times \vec{r} \right) \right]$ (From the vector identity).

1) $\vec{\Omega} \times \vec{r} = \vec{V}_t$ is the tangential velocity of a solid body rotation. As such, its divergence, $\nabla \bullet \left(\vec{\Omega} \times \vec{r} \right) = \nabla \bullet \vec{V}_t = 0$.

2) $\nabla \times \left(\vec{\Omega} \times \vec{r} \right) = \nabla \times \vec{V}_t$ is the vorticity, $\vec{\omega}$, of the solid body rotation. This is equal to $\vec{\omega} = 2\vec{\Omega}$, since the vorticity of solid-body rotation is just twice the angular velocity. The vorticity is also constant everywhere on the solid body. Therefore, the curl of the vorticity, $\nabla \times \vec{\omega}$, is zero.

Therefore, we've proven that $\nabla^2 \left(\vec{\Omega} \times \vec{r} \right) = 0$ and thus, $\nabla^2 \vec{V}_a = \nabla^2 \vec{V}_r$