ESCI 341 – Atmospheric Thermodynamics
Lesson 17 – Thermodynamic Diagrams

References: *Introduction to Theoretical Meteorology*, S.L. Hess, 1959
Glossary of Meteorology, AMS, 2000

GENERAL

- **Thermodynamic diagrams are used to graphically display the relation between two of the thermodynamic variables *T, V,* and *p*.**
- **Process lines represent specific thermodynamic processes on the diagram.**
- **Important process lines are**
  - **Isotherms**
  - **Isobars**
  - **Adiabats**
  - **Pseudoadiabats**
  - **Isohumes**
- **A useful thermodynamic diagram should have the following general properties**
  - **The area enclosed by a cyclic process should be proportional to the work done during the process.**
  - **As many of the process lines as possible should be straight.**
  - **The angle between the isotherms and adiabats should be as close to 90° as possible.**

CREATING A DIAGRAM WITH AREA PROPORTIONAL TO WORK

- **We've previously used the *p*-α diagram. Since work per unit mass is defined as**

$$dw = -pd\alpha$$

  the area on a *p*-α diagram is proportional to work.
- **The *p*-α diagram is not very useful for meteorologists because**
  - **The angle between isotherms and adiabats is very small**
  - **Process lines aren't very straight**
  - **Volume is not a convenient thermodynamic variable for meteorology**

- **For meteorology it would be useful to have a diagram that uses *T* and *p* as the thermodynamic variables.**
- **However, we can't just arbitrarily use *T* and *p* and hope that area will be proportional to work. We have to find a way of setting up the axes of our diagram so that area will be proportional to work.**
- **Let the variables for the axes be *X* and *Y*, and let *X* and *Y* be functions of the thermodynamic variables.**
- **For area to equal work in a cyclic process we need**

$$-\oint p\,d\alpha = \oint Y\,dX\ ,$$

or

$$\oint \left(p\,d\alpha + Y\,dX \right) = 0\,.$$

- **The quantity $pd\alpha$ + *YdX* is an exact differential. Therefore, from the Euler reciprocity relation**

$$\left(\frac{\partial p}{\partial X} \right)_{\alpha} = \left(\frac{\partial Y}{\partial \alpha} \right)_{X}. \qquad \textit{Equation A}$$

- **We are free to choose any *X* and *Y* and as long as they meet the above restriction then area will be proportional to work on our diagram.**

THE EMAGRAM

- **The *emagram* (energy-per-unit-mass diagram) is created by letting *X* = *T*. This means that**

$$\left(\frac{\partial p}{\partial T} \right)_{\alpha} = \left(\frac{\partial Y}{\partial \alpha} \right)_{T}.$$

- **Using the equation of state for dry air this becomes**

$$\left(\frac{\partial Y}{\partial \alpha} \right)_{T} = \frac{R_d}{\alpha}.$$

Integrating this with respect to α gives

$$Y = R_d \ln \alpha + F(T),$$

where *F*(*T*) is a completely arbitrary function of the integration.

From the ideal gas law, $\ln \alpha = -\ln p + \ln R_d + \ln T$,

so that

$$Y = -R_d \ln p + \left[R_d \ln R_d + R_d \ln T + F(T) \right].$$

Since *F*(*T*) is completely arbitrary, we can chose it to be

$$F(T) = -R_d \ln R_d - R_d \ln T\,,$$

so that

$$Y = -R_d \ln p\,.$$

- **For the emagram,**

$$X = T$$
$$Y = -R_d \ln p \quad .$$

- **On an emagram**
  - **Isotherms are straight vertical lines.**
  - **Isobars are straight, horizontal lines.**
  - **Adiabats are slightly curved, concave upward.**
  - **Pseudoadiabats are curved convex upward.**
  - **Isotherm – adiabat angle varies with axis scale, but is usually about 45°.**

TEPHIGRAM

- **Like the emagram, the tephigram is created by letting *X* = *T*, so that**

$$Y = R_d \ln\alpha + F(T)\,.$$

- **Instead of using the equation of state to find lnα, we use the definition of potential temperature**

$$\frac{\theta}{T} = \left(\frac{p_0}{p}\right)^{\kappa} = \left(\frac{p_0\alpha}{R_d T}\right)^{\kappa},$$

($\kappa = R_d/c_p$) so that

$$\ln\alpha = \frac{1}{\kappa}\ln\theta + \ln\left(\frac{R_d}{p_0}T^{\frac{\kappa-1}{\kappa}}\right).$$

This makes

$$Y = c_p \ln\theta + R_d \ln\left(\frac{R_d}{p_0}T^{\frac{\kappa-1}{\kappa}}\right) + F(T)\,.$$

If we let

$$F(T) = -R_d \ln\left(\frac{R_d}{p_0} T^{\frac{\kappa-1}{\kappa}}\right),$$

then

$$Y = c_p \ln\theta.$$

- **The tephigram gets its name from the fact that *Y* is proportional to entropy (which is sometimes given the symbol φ). Therefore its coordinates are *T* and φ, or *"tee-phi"*.**
- **On a tephigram**
  - **Adiabats are straight, horizontal lines**
  - **Isotherms are straight, vertical lines**
  - **The isotherm – adiabat angle is exactly 90°.**
  - **Isobars are logarithmic and slope upward to the right.**
- **When tephigrams are used, they are often rotated 45° clockwise so that the isobars are nearly horizontal, the adiabats slope upward toward the left, and the isotherms slope upward toward the right (similar to a skew-*T* log *p* diagram).**

SKEW-*T*/log *p* DIAGRAM

- **The skew-*T* log *p* diagram (or skew-*T* for short) is a modified emagram.**
- **As with the emagram, we will use**

$$Y = -R_d \ln p.$$

- **In order that area be proportional to work, we require**

$$\left(\frac{\partial p}{\partial X}\right)_\alpha = \left(\frac{\partial Y}{\partial \alpha}\right)_X,$$

- **Now**

$$\left(\frac{\partial Y}{\partial \alpha}\right)_X = -R_d\left(\frac{\partial(\ln p)}{\partial \alpha}\right)_X = -\frac{R_d}{p}\left(\frac{\partial p}{\partial \alpha}\right)_X = \frac{R_d}{\alpha}.$$

- **Also**

$$\left(\frac{\partial p}{\partial X}\right)_{\alpha} = -\frac{\left(\frac{\partial p}{\partial \alpha}\right)_{X}}{\left(\frac{\partial X}{\partial \alpha}\right)_{p}} = \frac{-p}{\alpha\left(\frac{\partial X}{\partial \alpha}\right)_{p}} \quad .$$

- **Therefore**

$$\left(\frac{\partial X}{\partial \alpha}\right)_{p} = -\frac{p}{R_d}.$$

- **If we integrate this with respect to α we get**

$$X = -\frac{p\alpha}{R_d} + f(p) = -T + f(p),$$

where $f(p)$ is any arbitrary function of p.

- **Since we can choose any function for $f(p)$, it makes sense to choose something proportional to the Y coordinate. We choose**

$$f(p) = K \ln p,$$

so that

$$X = -\frac{p\alpha}{R_d} + f(p) = -T + K \ln p$$

- **If we make one other slight change of coordinate so that $X' = -X$, then the coordinates for the skew-T can be written as**

$$X' = T - K \ln p$$
$$Y = -R_d \ln p \quad .$$

- **The equation for an isotherm is**

$$Y = -\frac{R_d}{K}T + \frac{R_d}{K}X'.$$

  - **The slope of the isotherms is given by R_d/K.**
  - **The slope, and the isotherm – adiabat angle can be varied by changing K, (or by changing the scale of the axes).**

- **On a skew-*T***
  - **Isobars are straight, horizontal lines**
  - **Isotherms are straight, sloping lines (toward the upper right).**
  - **Adiabats are nearly straight, sloping lines (toward the upper left).**

- **The isotherm – adiabat angle is nearly 90°.**

- ***Skew-T diagrams are the most commonly used thermodynamic diagrams in the U.S.***

STUVE DIAGRAM

- **Another fairly common thermodynamic diagram is the Stuve diagram.**
- **The Stuve diagram has the following axes**

$$X = T$$
$$Y = -p^{\kappa} \quad .$$

- **On a Stuve diagram**
    - **Isobars are straight, horizontal lines**
    - **Isotherms are straight, vertical lines**
    - **Adiabats are straight, sloping lines toward the upper left**
- **The Stuve diagram has two distinct disadvantages**
    - **The isotherm – adiabat angle is quite small**
    - **Area is not proportional to work**