ESCI 341 – Atmospheric Thermodynamics
Lesson 6 – Thermodynamic Processes

References: *An Introduction to Atmospheric Thermodynamics*, Tsonis
Introduction to Theoretical Meteorology, Hess
Physical Chemistry (4th edition), Levine
Thermodynamics and an Introduction to Thermostatistics, Callen

ISOTHERMAL PROCESSES

- **If a process is isothermal ($dT = 0$) then for an ideal gas the first law becomes**

$$dq = p d\alpha$$

- **For an ideal gas we can substitute for p from the ideal gas law to get**

$$dq = R'T \frac{d\alpha}{\alpha}$$

which integrates to

$$q = R'T \ln \frac{\alpha_f}{\alpha_i}.$$

- **We can also use the enthalpy form of the ideal gas law, which for an isothermal process becomes**

$$dq = -\alpha dp.$$

When this is integrated we get

$$q = -R'T \ln \frac{p_f}{p_i}.$$

ISOCHORIC PROCESSES

- **If a process is *isochoric* (constant volume) then the first law for an ideal gas becomes**

$$dq = c_v dT.$$

- **This can be integrated to get (assuming c_v is constant)**

$$q = c_v\left(T_f - T_i\right).$$

ISOBARIC PROCESSES

- **For an isobaric process, $dp = 0$. Therefore the first law for an ideal gas becomes**

$$dq = c_p dT$$

which integrates to

$$q = c_p\left(T_f - T_i\right).$$

ADIABATIC PROCESSES

- **An adiabatic process is one in which there is no heat transfer (*dq* = 0).**
- **The two forms of the first law of thermodynamics for an adiabatic process in an ideal gas are**

$$c_v dT = -p d\alpha$$
$$c_p dT = \alpha dp \quad .$$

- **If we start with the first form of the first law for an ideal gas (the one involving c_v) and substitute for pressure from the ideal gas law, we get**

$$c_v \frac{dT}{T} + R' \frac{d\alpha}{\alpha} = 0.$$

  - **Integrating this gives**

$$c_v \ln T + R' \ln \alpha = const.$$

    which can also be written as

$$T\alpha^{R'/c_v} = const. \qquad \textbf{(1)}$$

  - **We've previously shown that $c_p - c_v = R'$. Therefore, we can write Eqn. (1) as**

$$T\alpha^{(c_p - c_v)/c_v} = const.$$

    and defining the ratio $c_p/c_v \equiv \gamma$ we get

$$T\alpha^{\gamma-1} = const. \qquad \textbf{(2)}$$

  - **Using the ideal gas law, this equation can also be written as**

$$p\alpha^{\gamma} = const. \qquad \textbf{(3)}$$

    or

$$Tp^{(1-\gamma)/\gamma} = const. \qquad \textbf{(4)}$$

- **Equations (2), (3), and (4) are known as the *Poisson relations* (note that the constant on the right-hand-side is not necessarily the same in each equation.**

$$T\alpha^{\gamma-1} = const.$$
$$p\alpha^{\gamma} = const. \qquad \textbf{\textit{Poisson relations}}$$
$$Tp^{(1-\gamma)/\gamma} = const.$$

- **The Poisson relations relate T, p, and α in ideal gases undergoing *quasi-static, adiabatic processes*. If you know the initial values of two of these variables, and one of their final values, you can find the other two final values by using these relations.**
- ***It is important to realize that Poisson's relations are only valid for ideal gases undergoing quasi-static adiabatic processes!* It is inappropriate to use them for nonadiabatic processes.**

POTENTIAL TEMPERATURE

- ***Potential temperature* (denoted as θ) is defined as the temperature an air parcel would have if it were moved dry-adiabatically to a reference pressure, p_0, of 1000 mb.**
- **From the Poisson relation for T and p [Eqn. (4)] we get (see Exercise 11)**

$$\theta = T\left(\frac{p_0}{p}\right)^{R_d/c_p}.$$

- ***If an air parcel undergoes an adiabatic process its potential temperature is conserved.***

WORK IN AN ADIABATIC PROCESS

- **For an adiabatic process the change in internal energy is solely due to work done on or by the system, $du = dw$.**
  - ***Note that this is true for any system (not just ideal gasses) and regardless of whether the adiabatic process is quasi-static or not.***
- **For an ideal gas, $du = c_v dT = dw$.**

ADIABATIC FREE EXPANSION

- **If an ideal gas is allowed to adiabatically freely expand, unopposed, its temperature will not change.**
- **To see why, recall that for an ideal gas undergoing adiabatic expansion**

$$c_v dT = dw.$$

In a free expansion there is no work done, so there is no change in temperature.
- **But what about the expression**

$$dw = -pd\alpha\,?$$

The gas expanded, so specific volume changed, so shouldn't there be work accomplished?

KEY POINT: Remember that the expression $dw = -pd\alpha$ **only applies to quasi-static processes. A free expansion is not quasi-static, so we can't calculate work using this expression.**

EXERCISES

1. Show that for an isothermal process for an ideal gas

$$q = -R'T\ln\frac{p_f}{p_i}.$$

2. For an isothermal process for an ideal gas, show that the work done by the system is

$$w = -R'T\ln\frac{\alpha_f}{\alpha_i}$$

or

$$w = R'T\ln\frac{p_f}{p_i}.$$

3. a. For an isobaric process show that

$$\Delta u = c_p\left(T_f - T_i\right) + p\left(\alpha_i - \alpha_f\right).$$

b. Is this true for all gasses, or only ideal gasses?

4. Starting with $c_p dT = dq + \alpha dp$**, derive the Poisson relation** $Tp^{(1-\gamma)/\gamma} = const.$

5. A 1.5-kg parcel of dry air is at a temperature of 15°C and a pressure of 1013 mb.

a. How many moles of air are in the parcel? (The molecular weight of air is 28.96 g/mol)

b. What is the volume of the parcel?

c. What is the specific volume of the parcel?

d. If 50 KJ of heat are added to the parcel while its volume is held constant, what is the new temperature of the parcel? (The specific heat of air at constant volume is 717 J-kg^{-1}-K^{-1}).

6. An parcel of dry air is at a temperature of 15°C and a pressure of 1013 mb. Heat is added to the parcel to cause it to expand. It expands at constant pressure to 1.5 times its original volume.

a. What is the new temperature of the parcel?

b. How much work (per unit mass) was done by the parcel during this expansion?

c. What was the change in specific internal energy of the air parcel?

d. What was the amount of heat per unit mass that was added to the air parcel?

7. An air parcel is at a temperature of 15°C and a pressure of 1013 mb. Heat is added to the parcel to cause it to expand. It expands at constant temperature until its volume is 1.5 time it original volume.

a. What is the new pressure of the air parcel?

b. How much heat per unit mass was added to the air parcel?

c. How much work per unit mass was done in expanding the air parcel?

d. What was the change in specific internal energy of the air parcel?

8. A dry air parcel at an initial temperature of 20°C and a pressure of 950 mb is forced to rise adiabatically up a mountain slope. The top of the mountain is at a pressure of 720 mb.

a. What is the temperature of the air parcel when it reaches the top of the mountain?

b. What is the work done by the air parcel?

9. A cylinder filled with helium (a monatomic ideal gas) has a volume of 1.8×10^6 cm^3, a pressure of 1.2×10^5 mb, and a temperature of 300K. The cylinder is contained in an evacuated room with a volume of 16 m^3. The cylinder ruptures and helium fills the room.

a. What is the pressure in the room after the cylinder ruptures?

b. What is the temperature in the room after the cylinder ruptures?

c. What is the work done by the expanding helium?

10. A parcel of dry air is initially at a pressure of 900 mb and a temperature of 15°C. It rises to the 400 mb level.

a. What amount of heat (per mass) must be exchanged with its surroundings if the temperature is to remain constant at 15°C during the ascent? Will the heat be gained or lost by the parcel?

b. If the parcel first ascends adiabatically to 400 mb, and then heat is added to it to raise its temperature back to 15°C, how much heat must be added? Is this the same amount of heat as the previous question? If not, why not?

11. Using the Poisson relation

$$T p^{(1-\gamma)/\gamma} = const.$$

show that

$$\theta = T\left(p_0 / p\right)^{R_d/c_p}.$$

12. For a non-ideal gas, will an adiabatic free expansion result in a temperature change? Explain.