ESCI 341 – Atmospheric Thermodynamics
Answers to Selected Exercises for Lesson 6

3. a. For an isobaric process show that

$$\Delta u = c_p\left(T_f - T_i\right) + p\left(\alpha_i - \alpha_f\right).$$

Answer: Start with the definition of enthalpy,

$$h = u + p\alpha \,.$$

Differentiate to get

$$dh = du + \alpha dp + pd\alpha \,.$$

For an isobaric process this is

$$dh = du + pd\alpha \,.$$

Rearrange to get

$$du = dh - pd\alpha$$

For an ideal gas we know that $dh = c_p dT$

So we have

$$du = c_p dT - pd\alpha \,.$$

Integrating gives

$$\Delta u = c_p\left(T_f - T_i\right) + p\left(\alpha_i - \alpha_f\right).$$

b. Is this true for all gasses, or only ideal gasses?

Answer: Ideal gasses only, since we used $dh = c_p dT$.

5. A 1.5-kg parcel of dry air is at a temperature of 15°C and a pressure of 1013 mb.

a. How many moles of air are in the parcel? (The molecular weight of air is 28.96 g/mol) **Answer: 51.80 mol**

b. What is the volume of the parcel? **Answer: 1.22 m^3**

c. What is the specific volume of the parcel? **Answer: 0.82 m^3/kg**

d. If 50 KJ of heat are added to the parcel while its volume is held constant, what is the new temperature of the parcel? (The specific heat of air at constant volume is 717 J-kg^{-1}-K^{-1}). **Answer: 61.5°C**

6. An parcel of dry air is at a temperature of 15°C and a pressure of 1013 mb. Heat is added to the parcel to cause it to expand. It expands at constant pressure to 1.5 times its original volume.

   a. What is the new temperature of the parcel? **Answer: 159°C**

   b. How much work (per unit mass) was done by the parcel during this expansion?
      Answer: 41.5 kJ/kg

   c. What was the change in specific internal energy of the air parcel?
      Answer: 103 kJ/kg

   d. What was the amount of heat per unit mass that was added to the air parcel?
      Answer: 144.6 kJ/kg

7. An air parcel is at a temperature of 15°C and a pressure of 1013 mb. Heat is added to the parcel to cause it to expand. It expands at constant temperature until its volume is 1.5 time it original volume.

   a. What is the new pressure of the air parcel? **Answer: 675.3 mb**

   b. How much heat per unit mass was added to the air parcel? **Answer: 33.5 kJ/kg**

   c. How much work per unit mass was done in expanding the air parcel?
      Answer: –33.5 kJ/kg

   d. What was the change in specific internal energy of the air parcel? **Answer: 0**

8. A dry air parcel at an initial temperature of 20°C and a pressure of 950 mb is forced to rise adiabatically up a mountain slope. The top of the mountain is at a pressure of 720 mb.

   a. What is the temperature of the air parcel when it reaches the top of the mountain?

      Answer: −2°C

   b. What is the work done by the air parcel? **Answer: −15.8 kJ/kg**

9. A cylinder filled with helium (a monatomic ideal gas) has a volume of 1.8×10^6 cm^3, a pressure of 1.2×10^5 mb, and a temperature of 300K. The cylinder is contained in an evacuated room with a volume of 16 m^3. The cylinder ruptures and helium fills the room.

   a. What is the pressure in the room after the cylinder ruptures? **Answer: 13,500 mb**

   b. What is the temperature in the room after the cylinder ruptures? **Answer: 300K**

   c. What is the work done by the expanding helium? **Answer: none (in a free expansion, there is no opposing force, so no work is done)**

10. A parcel of dry air is initially at a pressure of 900 mb and a temperature of 15°C. It rises to the 400 mb level.

   a. What amount of heat (per mass) must be exchanged with its surroundings if the temperature is to remain constant at 15°C during the ascent? Will the heat be gained or lost by the parcel? **Answer: 67.1 kJ/kg gained**

   b. If the parcel first ascends adiabatically to 400 mb, and then heat is added to it to raise its temperature back to 15°C, how much heat must be added?
      Answer: 59.9 kJ/kg