ESCI 341 – Atmospheric Thermodynamics
Answers to Selected Exercises for Lesson 3

1. Prove that for a volume of arbitrary shape that the work done in expanding the volume by a differential volume, dV, is pdV. Hint: Imagine every where on the surface of the volume that the surface is pushed out an amount $d\vec{n}$, where $\vec{n}$ is a vector whose direction is everywhere normal to the surface.

Answer: If the surface area of the original volume is *A*, and the surface is expanded a distance *dn* normal to the surface, the differential volume, *dV*, is

$$dV = Adn.$$

The work done by a unit of surface area during the expansion is

$$dW_{\delta A} = \frac{dW}{A} = \frac{\vec{F} \bullet d\vec{n}}{A} = p\,dn \qquad (1)$$

where the subscript δA reminds us that this is for a small unit of surface area. From Eq. (1) we can write

$$dW = p\,Adn = pdV\,. \qquad (2)$$

2. a. What is the minimum amount of work done by you in blowing up a spherical party balloon to a diameter of 8 inches? Assume standard sea-level pressure.

Answer: The total work done in blowing up the balloon is

$$W = \int pdV = p\Delta V = p\left(V_f - V_0\right) = \frac{\pi}{6} pd^3,$$

where we assume the balloon began with zero initial volume, and *d* is the final diameter. For a pressure of 101325 Pa and a final diameter of 0.20 m, the work is 445 J.

b. Why is this the minimum amount of work?

Answer: We've neglected the work done in stretching the latex that the balloon is made of.