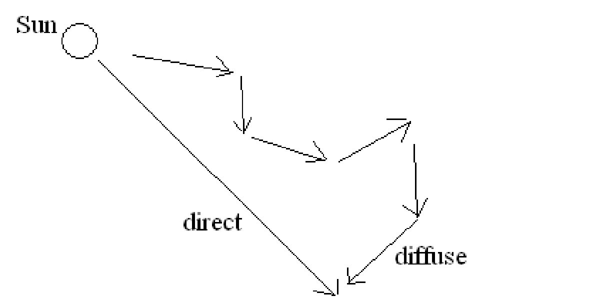
ESCI 340 – Physical Meteorology
Radiation Lesson 7 – Solar Radiation

References: *An Introduction to Atmospheric Radiation*, Liou
Radiation and Cloud Processes in the Atmosphere, Liou
Atmospheric Science: An Introductory Survey, Wallace and Hobbs

Reading: Petty, Sections 7.4.1 through 7.4.3

DIRECT AND DIFFUSE RADIATION

- **We categorize solar radiation as either**
  - ***Direct* – This radiation comes directly from the Sun without scattering or absorption.**
  - ***Diffuse* – This is solar radiation that has been scattered at least once.**



THE SOLAR CONSTANT

- **The *solar constant*, S_0, is defined as the total solar flux at the top of the Earth's atmosphere on a surface perpendicular to the Sun's rays at a distance of the Earth's mean distance to the sun (1 AU).**
- **The solar constant is really not constant…it varies with the power output from the Sun itself, which isn't constant.**
  - **The solar constant is around 1368 W/m^2.**
- **Since the flux at the top of the atmosphere varies inversely with the square of the distance from the sun, d, the solar constant, S_0, is related to S via**

$$S_0 = S\left(\frac{d}{d_m}\right)^2, \quad (1)$$

where d_m is the mean distance of Earth from the Sun (1 AU).

  - **The distance to the Sun can be found from**

$$\frac{d}{d_m} = \frac{1-\varepsilon^2}{1-\varepsilon\cos(\alpha n)} \quad \textbf{(2)}$$

where ε is the eccentricity of the Earth's orbit (~.0167), n is the number of days from aphelion, and α is the orbital rate of the Earth (~0.9863°/day).

INSOLATION

- ***Insolation* is defined as the solar irradiance upon a horizontal area (not necessarily perpendicular to the Sun's rays).**
- **Insolation is a contraction for *IN*coming *SOL*ar radi*ATION*.**
- **The solar insolation, F_S, at the top of the atmosphere is the total solar flux (perpendicular to the solar rays) multiplied by the cosine of the zenith angle,**

$$F_S = S\cos\theta = S_0\left(\frac{d_m}{d}\right)^2\cos\theta\,. \quad \textbf{(3)}$$

- **The insolation integrated with time over the hours of daylight will give the energy per unit area per day received at a point on the Earth.**
- **The relationship between the insolation and the total energy per unit area (Q) is**

$$F_S = dQ/dt\,, \quad \textbf{(4)}$$

so that

$$Q = \int F_S\,dt\,. \quad \textbf{(5)}$$

- **The total energy per unit area per day received at the top of the atmosphere is then found by**

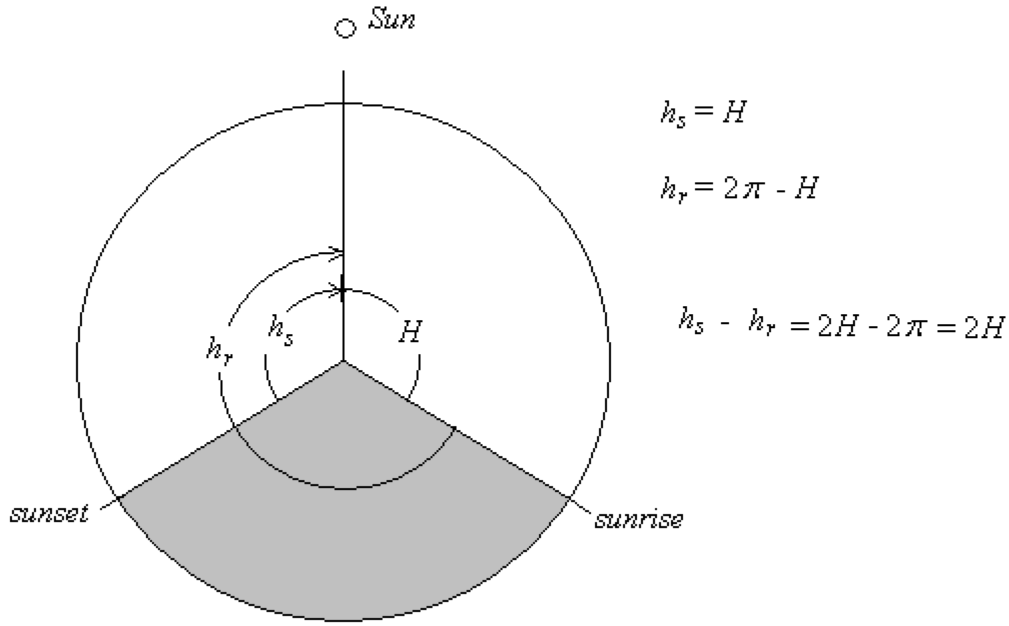
$$Q_\infty = \int_{t_r}^{t_s} S\,dt = S_0\left(\frac{d_m}{d}\right)^2\int_{t_r}^{t_s}\cos\theta(t)\,dt \quad \textbf{(6)}$$

where t_r is the time of sunrise, and t_s is the time of sunset.

- **From the expression for zenith angle that we had in a prior lesson, (6) becomes**

$$Q_\infty = S_0\left(\frac{d_m}{d}\right)^2\int_{t_r}^{t_s}\left(\sin\lambda\sin\delta + \cos\lambda\cos\delta\cosh h\right)dt\,. \quad \textbf{(7)}$$

- **The diagram below shows a latitude circle. The shaded region represents the nighttime portion of the latitude circle. The hour angle at sunset is $h_s = H$, which is also called the *half-day angle*. The hour angle at sunrise is $h_r = 2\pi - H$.**



- **The angular velocity of the Earth is Ω = *dh/dt* so that we can substitute *dt = dh/*Ω into (7) to get**

$$Q_\infty = \frac{S_0}{\Omega}\left(\frac{d_m}{d}\right)^2 \int_{h_r}^{h_s} \left(\sin\lambda\sin\delta + \cos\lambda\cos\delta\cos h\right) dh \,. \qquad \textbf{(8)}$$

which when integrated results in

$$Q_\infty = \frac{S_0}{\Omega}\left(\frac{d_m}{d}\right)^2 \left[\left(h_s - h_r\right)\sin\lambda\sin\delta + \cos\lambda\cos\delta\left(\sin h_s - \sin h_r\right)\right]. \qquad \textbf{(9)}$$

From the relations

$$h_s - h_r = 2H - 2\pi = 2H \qquad \textbf{(10)}$$

and

$$\sin h_s - \sin h_r = \sin H - \sin(2\pi - H) = \sin H + \sin H = 2\sin H \qquad \textbf{(11)}$$

equation (9) finally becomes

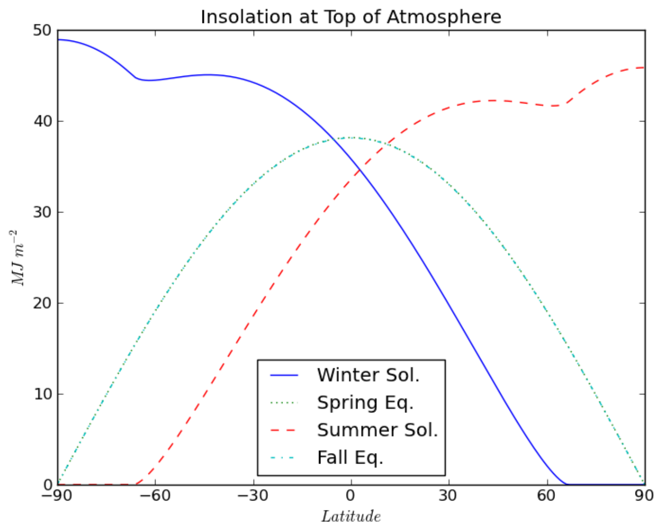
$$Q_\infty = \frac{2S_0}{\Omega}\left(\frac{d_m}{d}\right)^2 \left[H\sin\lambda\sin\delta + \cos\lambda\cos\delta\sin H\right]. \qquad \textbf{(12)}$$

- **The half-day angle, *H*, must have units of radians in the first term of the above formula.**

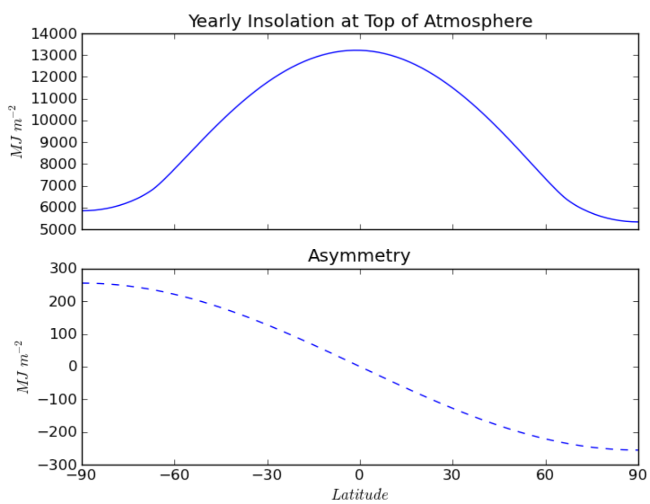
- **The half-day angle is a function of latitude and declination, and is found from**

$$\cos H = -\tan\lambda \tan\delta\,. \quad \textbf{(13)}$$

- **The figure below shows a plot of the total energy per area, Q, versus latitude at the top of the atmosphere for the Equinoxes and the Solstices.**

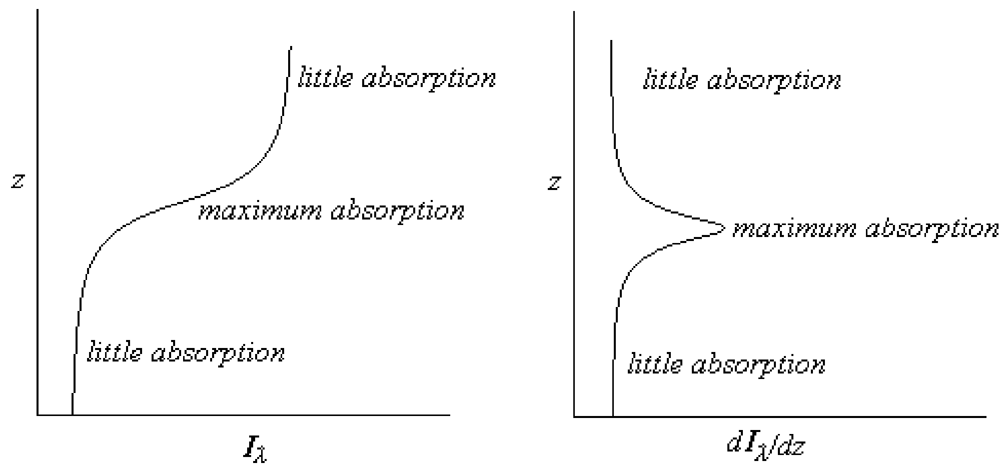


- **The figure below shows the sum of the total energy per area, Q, versus latitude at the top of the atmosphere for an entire year. The bottom plot shows the asymmetric part of *Q*, and shows that the Southern Hemisphere receives more total radiation than does the Northern Hemisphere.**



VERTICAL PROFILE OF ABSORPTION

- **When solar radiation is passing through the atmosphere, some of it is absorbed and heats the atmosphere.**
  - **At the levels where absorption is occurring,** $dI_\lambda / dz > 0$.
  - **At levels where there is little absorption,** dI_λ / dz **will be very small.**
  - **The diagram below shows possible profiles of** I_λ **and** dI_λ / dz **and their relationship to absorption.**



- **The transmittance of solar radiation from the top of the atmosphere (**$z = \infty$**) to some level** z **is given by**

$$t_\lambda(z) = I_\lambda(z) / I_\lambda(\infty) \quad \textbf{(14)}$$

where $I_\lambda(z)$ **is the intensity at level** z **and** $I_\lambda(\infty)$ **is the monochromatic intensity at the top of the atmosphere. Differentiating (14) with respect to** z **shows that**

$$\frac{dt_\lambda}{dz} \propto \frac{dI_\lambda}{dz} \quad \textbf{(15)}$$

so we can also say that if $dt_\lambda / dz > 0$ **then there is absorption taking place at level** z**.**

  - **The larger** dt_λ / dz **the greater the absorption.**
  - **If** dt_λ / dz **is zero then there is no absorption.**

- **Since we know from Beer's Law that**

$$t_\lambda = \exp\left(-\tau_{d\lambda} / \mu\right) \quad \textbf{(16)}$$

then the measure of absorption is given as

$$\frac{dt_\lambda}{dz} = -\frac{1}{\mu}\frac{d\tau_{d\lambda}}{dz}\exp\left(-\tau_{d\lambda}/\mu\right), \tag{17}$$

We also know from Lesson 4 that

$$\frac{d\tau_{d\lambda}}{dz} = -\rho k_\lambda \tag{18}$$

so the measure of absorption is

$$\frac{dt_\lambda}{dz} = \frac{\rho k_\lambda}{\mu}\exp\left(-\tau_{d\lambda}/\mu\right). \tag{19}$$

- **If we want to find at what optical depth the absorption will be maximum, we need to take the derivative of (19) with respect to optical depth and set it equal to zero. If we do this we get**

$$\frac{d\rho}{dz} + \frac{k_\lambda}{\mu}\rho^2 = 0. \tag{20}$$

- **Equation (20) tells us that the maximum absorption will occur at the altitude where the vertical gradient of density, $\partial\rho/\partial z$, is equal to $-\left(k_\lambda/\mu\right)\rho^2$. This altitude will depend on the density distribution of the atmosphere.**

LEVEL OF MAXIMUM ABSORPTION IN AN EXPONENTIAL ATMOSPHERE

- **To a reasonable approximation the vertical density profile of Earth's atmosphere is given by**

$$\rho(z) = \rho_0 \exp\left(-z/H\right), \tag{21}$$

where *H* is called the *scale height* of the atmosphere and is given by

$$H = R_d T/g \tag{22}$$

where *T* is a representative average temperature.

- **In an exponential atmosphere the optical depth is (see exercises)**

$$\tau_{d\lambda} = k_\lambda \rho H. \tag{23}$$

- **Putting (21) into (20) results in**

$$k_\lambda \rho H = \mu, \tag{24}$$

which from (23) is just

$$\tau_{d\lambda} = \mu. \tag{25}$$

- **Equation (25) gives us the very important result that in a plane-parallel atmosphere with an exponential decay of density with altitude, the level of maximum absorption of solar radiation will occur at that level where the optical depth is equal to the cosine of the zenith angle.**
- **The path optical thickness from the top of the atmosphere to some level *z* is related to the optical depth via**

$$\tau_{s\lambda} = \tau_{d\lambda}/\mu , \qquad \textbf{(26)}$$

so we can also state that *<u>in a plane-parallel atmosphere that has an exponential decay of density with altitude, the level of maximum absorption will occur at that altitude where the path optical thickness from the top of the atmosphere is equal to one</u>*.

- ***<u>A word of caution</u>*: In the above analysis we've assumed that the molecules causing the extinction are well mixed. This is not always the case, particularly for the variable gasses.**

SOLAR HEATING RATES

- **Solar radiation at any level in the atmosphere consists of a diffuse component and a direct (parallel beam) component.**
- **At any level in the atmosphere there is radiation arriving from both the upward and downward directions. Denoting the downward fluxes with a minus sign, and upward fluxes with a plus sign, we have**

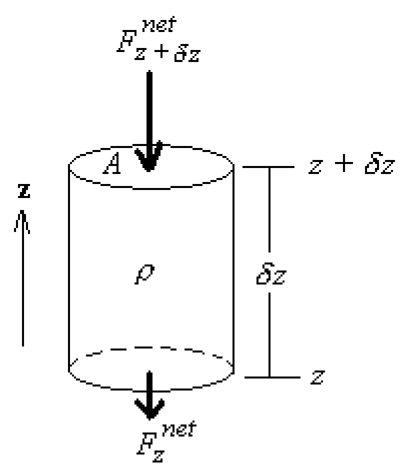
$$\begin{aligned} F^- &= F^-_{direct} + F^-_{diffuse} \\ F^+ &= F^+_{diffuse} \end{aligned} \qquad \textbf{(27)}$$

    - **Note: Remember that *flux* and *irradiance* are the same thing. We are using the flux notation here because physically it is more intuitive, but we could just as well called this the upward and downward irradiances.**
    - **Also, keep in mind that we are only dealing with solar fluxes (solar radiation) in this section. We are not accounting for any radiation emitted by the Earth or atmosphere.**

- **The net flux at any level is**

$$F_{net}(z) = F^{-}(z) - F^{+}(z)\,. \tag{28}$$

  - **The reason the net flux is defined as the downward minus the upward, and not the other way, is that the downward flux is likely to be larger than the upward flux, and we would like the net flux to be a positive number. It could be defined the other way, and as long as we were careful everything would still work out.**
  - ***F_{net}* is positive if downward, and negative if upward.**
- **Now imagine a cylindrical air parcel (it could be any shape, but cylinders are easy to work with) as shown below.**



- **The total power entering the cylinder from both the top and bottom is**

$$P = F^{net}_{z+\delta z}A - F^{net}_{z}A\,. \tag{29}$$

- **The heating rate is proportional to the power absorbed, via**

$$\frac{dT}{dt} = \frac{dT}{dQ}\frac{dQ}{dt} = \frac{dT}{dQ}P\,, \tag{30}$$

where *Q* is the energy absorbed (so *dQ /dt = P*). From the definition of heat capacity, *C_p = dQ /dT,* this becomes

$$\frac{dT}{dt} = \frac{1}{C_p}P = \frac{A}{C_p}\left(F^{net}_{z+\delta z} - F^{net}_{z}\right). \tag{31}$$

- **We also can write the heat capacity in terms of specific heat, area, and *δz* as**

$$C_p = \rho V c_p = \rho A \delta z\, c_p\,, \tag{32}$$

so that

$$\frac{dT}{dt} = \frac{1}{\rho c_p} \frac{F_{z+\delta z}^{net} - F_z^{net}}{\delta z}. \quad \textbf{(33)}$$

- **In the limit as $\delta z \rightarrow 0$, then**

$$\lim_{\delta z \to 0} \frac{F_{z+\delta z}^{net} - F_z^{net}}{\delta z} \equiv \frac{dF^{net}}{dz} \quad \textbf{(34)}$$

(this is nothing more than the definition of a derivative from the First Fundamental Theorem of Calculus.) Therefore, we have the result that the heating rate is directly proportional to the derivative of the net flux with height,

$$\frac{dT}{dt} = \frac{1}{\rho c_p} \frac{dF^{net}}{dz}. \quad \textbf{(35)}$$

- **Equation (35) is important because this is how we can calculate the effects of solar and terrestrial radiation on the atmosphere, and determine whether the atmosphere at altitude *z* will be warming or cooling due to radiation.**

EXERCISES

1. Calculate the difference in solar radiation at the top of the atmosphere between perihelion and aphelion.

2. a. Find the amount of energy received per square meter in one day at the top of the atmosphere at the latitudes and times in the table below. Assume that $d = d_m$.

Date	Latitude	Energy per area per day
Equinox	**0**	
	60N	
	90N	
Summer Solstice	**90N**	
	0	
	90S	

b. At the Summer Solstice, the sun angle is higher at the equator than at the North Pole, and yet the maximum in energy is at the North Pole. Why?

3. a. Show that by differentiating

$$\frac{dt_\lambda}{dz} = \frac{\rho k_\lambda}{\mu} \exp\left(-\tau_{d\lambda}/\mu\right)$$

with respect to *z* and setting it equal to zero that it results in

$$\frac{d\rho}{dz} + \frac{k_\lambda}{\mu}\rho^2 = 0 .$$

b. Using the definition of optical depth from Lesson 4, show that in an exponential atmosphere the optical depth is

$$\tau_{d\lambda} = k_\lambda \rho H .$$

c. Put the density profile for an exponential atmosphere (equation (2)) into equation (1) above to show that it results in

$$k_\lambda \rho H = \mu .$$

d. Show that in an exponential atmosphere that the altitude of maximum absorption is given by

$$z_{\max} = H \ln\left(k_\lambda H \rho_0 / \mu\right)$$

4. UV radiation of wavelength 0.25μm penetrates to an altitude of about 45 km above the surface of the Earth.

a. Estimate the mass-extinction coefficient using the equation for z_{max} above. Use a temperature of 288 K when calculating the scale height, and assume a sea-level density of air of 1.23 kg/m^3. Also, assume a zenith angle of zero.

b. Radiation at 0.25μm is absorbed primarily by ozone in the stratosphere. Do you think your estimate of extinction coefficient in part a. is very accurate? If not, explain why not.

5. Using the data in the table below, find the solar heating rates (in °C/day) at 5, 15, 25, and 35 km. You can find density by assuming a scale height of 8.1 km and a surface density of 1.23 kg/m^3. Use c_p = 1006 J-kg^{-1}-K^{-1}.

Altitude (km)	F_{net} (W/m^2)
0	275
10	335
20	345
30	349
40	350