ESCI 340 – Physical Meteorology
Radiation Lesson 5 – Terrestrial Radiation and Radiation Balance
Dr. DeCaria

References: *Atmospheric Science: An Introductory Survey*, Wallace and Hobbs
An Introduction to Atmospheric Radiation, Liou
A First Course in Atmospheric Radiation, Petty

Reading: Petty, Chapter 8 (don't focus on equations in book)

SCHWARZSCHILD'S EQUATION

- **The radiative transfer equation is**

$$dI_\lambda = -k_\lambda \rho I_\lambda ds + k_\lambda \rho J_\lambda ds.$$

- **If we make the following assumptions:**
  - **Ignore multiple scattering (so there is no scattering back into the beam)**
  - **Assume the medium is a blackbody, so the source function is the Planck function for radiance, B_λ.**

  then the radiative transfer equation becomes

$$dI_\lambda = -k_\lambda \rho \left(I_\lambda - B_\lambda \right) ds.$$

- **The equation above is known as *Schwarzschild's equation*.**
- **We would like to apply Schwarzschild's equation to a plane parallel atmosphere, for both upward and downward propagating radiation.**
  - **For downward propagating radiation**

$$ds = -dz/\mu$$

  so that Schwarzschild's equation is

$$dI_\lambda = \frac{k_\lambda \rho \left(I_\lambda - B_\lambda \right)}{\mu} dz. \quad \textit{Downward propagating radiation}$$

  - **For upward propagating radiation**

$$ds = dz/\mu$$

  and the Schwarzschild equation is

$$dI_\lambda = -\frac{k_\lambda \rho \left(I_\lambda - B_\lambda \right)}{\mu} dz. \quad \textit{Upward propagating radiation}$$

EMISSION FROM AN OPTICALLY THICK LAYER

- **In general, the Schwarzschild equation does not have an analytical solution. However, it is instructive to solve it for the case of an isothermal atmosphere, in which case B_λ = constant.**
- **For upward propagating radiation the intensity at some level *z* in the atmosphere is found by integrating Schwarzschild's equation from the ground to level *z*,**

$$\int_{I_{0\lambda}}^{I_\lambda} \frac{dI_\lambda}{I_\lambda - B_\lambda} = -\int_0^z \frac{k_\lambda \rho}{\mu} dz \,.$$

- **The Planck function, B_λ, depends on the temperature, so the left side can't easily be integrated. However, if we assume an isothermal atmosphere then B_λ is constant, and the integral is**

$$I_\lambda(z) - B_\lambda = \left(I_{\lambda 0} - B_\lambda\right)\exp(-\tau_{z\lambda}/\mu),$$

  where $\tau_{z\lambda}$ is the vertical optical thickness from the ground to level *z*.
- **From this result, we see that as the optical thickness of a layer increases, then $I_\lambda(z)$ approaches B_λ. Thus, an optically thick layer behaves as though all the radiation were being emitted by the upper surface of the layer.**
- **Though this result was derived in terms of an isothermal layer, in general it is true for any optically thick layer. This is why clouds appear cold on an IR satellite image, because the satellite is only seeing the radiation emitted by the cloud top, not all the radiation that was emitted at higher temperatures below the cloud top.**

GLOBAL RADIATION BALANCE

- **The amount of solar power received by the entire Earth is**

$$P_{in} = \pi R^2 (1-\alpha) S$$

  where *R* is the radius of the Earth, α is the albedo, and *S* is the solar constant.
- **The amount of power radiated by the Earth is**

$$P_{out} = 4\pi R^2 \sigma T_e^4,$$

  where T_e is the temperature at which the Earth/atmosphere system is radiating (σ is the Stefan-Boltzmann constant).

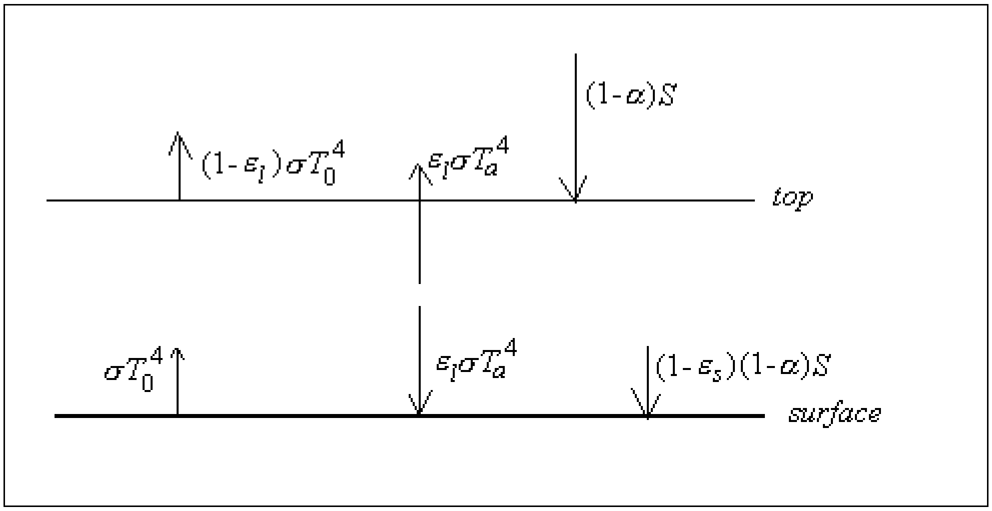
- **If the Earth's temperature is not changing, then the power in must equal the power out. This leads to an expression for the radiation temperature,**

$$T_e = \sqrt[4]{\frac{(1-\alpha)S}{4\sigma}} \; .$$

- **For the Earth, the radiation temperature is approximately 255 K (–18°C), which is the temperature at about 500 mb.**
- **The surface temperature of the Earth averages about 288 K (15°C). The reason the surface temperature is much warmer than the radiation temperature is that the atmosphere contains greenhouse gasses.**
    - **On a planet without an atmosphere, the radiation and surface temperatures would be identical.**

RADIATION BALANCE AND THE GREENHOUSE GASSES

- **To further illustrate the role of greenhouse gasses, imagine an atmosphere that consists of a single, homogeneous slab such as that shown below.**



- **In this model, the atmosphere has an albedo of α. The surface of the planet is a blackbody at a temperature of T_0. The atmosphere is a gray body, with an absorptivity of ε_s for short-wave radiation and an absorptivity of ε_l for long-wave radiation (from Kirchhoff's law, the absorptivity and emissivity are equal). The temperature of the atmosphere is T_a.**

- **We can write a set of equations for the radiation balance at the top of the atmosphere and at the surface.**
  - **At the top of the atmosphere:**
    - **Incoming solar radiation (after reflection) is $(1-\alpha)S$**
    - **Radiation emitted by the atmosphere and escaping to space is $\varepsilon_l \sigma T_a^4$**
    - **Radiation emitted by the surface and escaping to space is $(1-\varepsilon_l)\sigma T_0^4$ (note that absorptivity equals emissivity, so the amount absorbed by the atmosphere is $\varepsilon_l \sigma T_0^4$, leaving one minus this amount to escape to space.**
    - ***Radiation balance at top of atmosphere*:**

$$(1-\alpha)S_0 - (1-\varepsilon_l)\sigma T_0^4 - \varepsilon_l \sigma T_a^4 = 0 \qquad \textbf{(1)}$$

  - **At the surface:**
    - **Incoming solar radiation is $(1-\alpha)(1-\varepsilon_s)S_0$ (note that absorptivity equals emissivity, so the amount absorbed by the atmosphere is $(1-\alpha)\varepsilon_s S$, leaving one minus this amount to reach the surface.)**
    - **Incoming radiation emitted by atmosphere is $\varepsilon_l \sigma T_a^4$**
    - **Outgoing radiation emitted by surface is σT_0^4**
    - ***Radiation balance at surface*:**

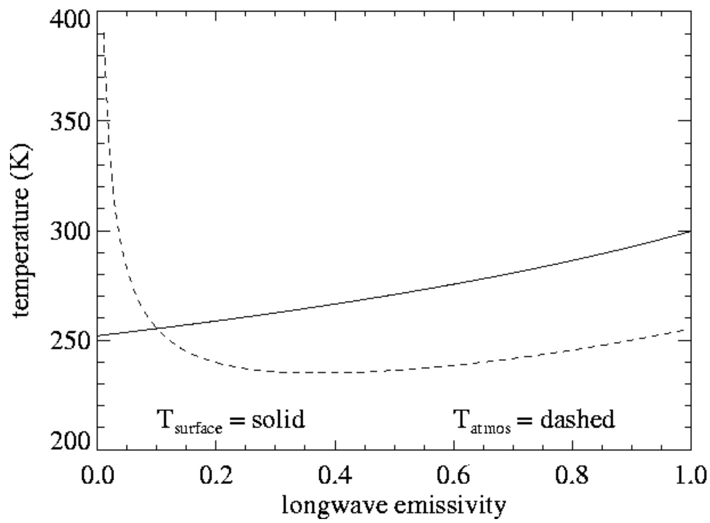
$$(1-\alpha)(1-\varepsilon_s)S_0 + \varepsilon_l \sigma T_a^4 - \sigma T_0^4 = 0 \qquad \textbf{(2)}$$

- **Equations (1) and (2) can be solved for T_a and T_0, with the following results:**

$$T_a = \sqrt[4]{\frac{(\varepsilon_l - \varepsilon_s\varepsilon_l + \varepsilon_s)(1-\alpha)S}{\varepsilon_l(2-\varepsilon_l)\sigma}}$$

$$T_0 = \sqrt[4]{\frac{(2-\varepsilon_s)(1-\alpha)S}{(2-\varepsilon_l)\sigma}}.$$

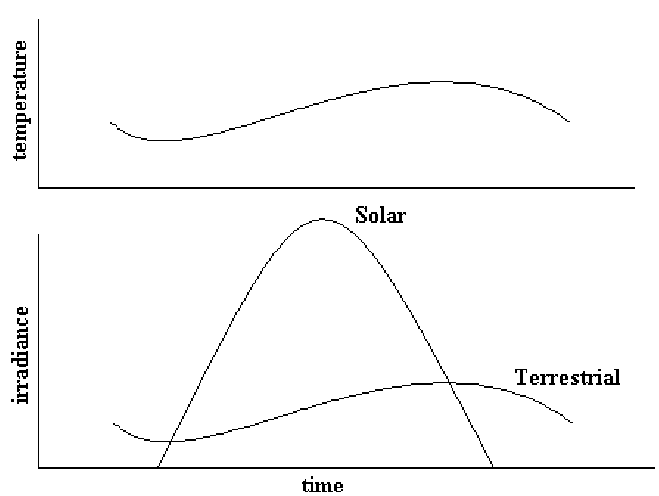
- **The figure below shows a plot of these temperatures as a function of ε_l (for this plot ε_s is set to 0.1 and S = 344 W m^2).**



- **An increase in greenhouse gasses results in an increase of ε_l, and therefore, a monotonic increase in the surface temperature, T_0.**
- **The effect of the greenhouse gasses on the atmospheric temperature is not monotonic.**
  - **At first, increasing greenhouse gasses actually decreases atmospheric temperature, while later it increases.**
- **Though the real atmosphere is certainly more complex than the simple slab model, the slab model remains a useful illustration.**

DAILY AND YEARLY RADIATION BALANCE

- **A location on Earth receives its maximum solar radiation at local noon, yet the hottest time of day is in the afternoon. This apparent discrepancy can be explained in terms of radiation balance.**

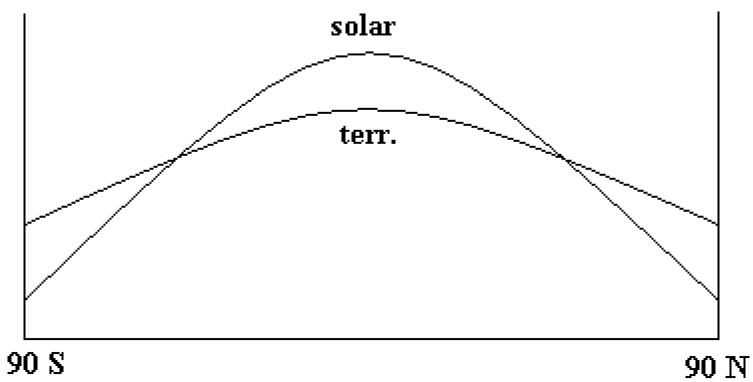


- **During those times when the outgoing radiation exceeds the incoming, the temperature falls.**
- **When the incoming radiation exceeds the outgoing, the temperature rises.**
- **When the two components of radiation are equal, there is no change in temperature.**

- **A similar concept applies to the annual cycle, where the maximum incoming solar radiation occurs at the Solstice (June in Northern Hemisphere), but the hottest month of the year is actually later in the summer (July or August in Northern Hemisphere).**

LATITUDINAL RADIATION BALANCE

- **At the top of the atmosphere, the poles actually receive more insolation throughout the year than does the Equator. However, at the surface of the Earth the Equatorial regions receive far more insolation.**
- **A sketch of the solar and terrestrial radiation fluxes vs. latitude looks like**



- **The Tropical regions receive more energy than they radiate, and so should become increasingly hotter.**
- **The Polar regions radiate more energy than they receive, and so should become increasingly colder.**
- **Therefore, the atmosphere and oceans must somehow transport the excess heat from the Tropics to the Poles.**
    - **This is what ultimately drives the circulation of the atmosphere and oceans.**

EXERCISES

1. **Find the radiation temperature of the Earth for a solar constant of 1373 W/m^2 and an albedo of 30%.**

2. **Why is this temperature so much less than the surface temperature?**

3. **If the albedo increased, would the radiation temperature increase or decrease?**

4. **If the solar constant increased, would the radiation temperature increase or decrease?**

5. **If the Earth became cloudier, would the radiation temperature increase or decrease? What about the surface temperature?**

6. **For the slab model**

   a. **show that**

$$T_a = \sqrt[4]{\frac{(\varepsilon_l - \varepsilon_s \varepsilon_l + \varepsilon_s)(1-\alpha)S}{\varepsilon_l (2-\varepsilon_l)\sigma}}.$$

   and

$$T_0 = \sqrt[4]{\frac{(2-\varepsilon_s)(1-\alpha)S}{(2-\varepsilon_l)\sigma}}.$$

   b. **Using the values in the table below, find T_0 and T_a.**

	value
S	**344 $W\text{-}m^{-2}$**
α	**0.3**
ε_s	**0.1**
ε_l	**0.8**

   c. **Explain physically why, if there are very few greenhouse gasses, the temperature of the atmosphere in the slab model gets extremely large.**