**ESCI 340 – Physical Meteorology**
**Radiation Lesson 2 – Radiation Laws**
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**References:** *Atmospheric Science: An Introductory Survey*, Wallace and Hobbs
*An Introduction to Atmospheric Radiation*, Liou
*A First Course in Atmospheric Radiation*, Petty

**Reading:** Petty, Chapter 6 (up through Section 6.3)

## PLANCK'S LAW

- **A *blackbody* is a perfect emitter and absorber of radiation.**
- **The monochromatic intensity emitted by a blackbody is described by the Planck function**

$$B_\lambda = \frac{2hc^2}{\lambda^5\left[\exp\left(hc/k\lambda T\right)-1\right]}.$$

  - ***k* is the Boltzmann constant, and is $1.38\times10^{-23}$ J-K$^{-1}$**
  - ***h* is Planck's constant and is $6.626\times10^{-34}$ J-s**
  - ***c* is the speed of light in a vacuum and is $2.9979\times10^{8}$ m s$^{-1}$.**
- **NOTE: $B_\lambda$ is monochromatic intensity, and has units of W m$^{-2}$ sr$^{-1}$ $\mu$m$^{-1}$**
- **Blackbody radiation is isotropic.**
  - **In a prior lesson we showed that for isotropic radiation that flux and intensity are related by**

$$F = \pi I.$$

    **Therefore, the monochromatic flux emitted by a blackbody is**

$$F_{BB\lambda} = \frac{2\pi hc^2}{\lambda^5\left[\exp\left(hc/k\lambda T\right)-1\right]},$$

    **where the subscript *BB* reminds us that this is for a blackbody**
  - **If we define two new constants, $c_1 = 2\pi hc^2$ and $c_2 = hc/k$ then Planck's law can be written in a more concise way as**

$$F_{BB\lambda} = \frac{c_1}{\lambda^5\left[\exp\left(c_2/\lambda T\right)-1\right]}.$$

**WIEN'S DISPLACEMENT LAW**

- **Wien's displacement law gives the wavelength of maximum emission for a blackbody.**
- **Wien's displacement law is found by setting**

$$\frac{\partial F_{BB\lambda}}{\partial \lambda} = 0$$

**and solving for $\lambda$ (see exercises) to get**

$$\lambda_{\max} = \frac{2897\ \mu m\ K}{T}.$$

**STEFAN-BOLTZMANN LAW**

- **The Stefan-Boltzmann law gives the total flux emitted from a blackbody. It is found by**

$$F_{BB} = \int_0^\infty \frac{c_1}{\lambda^5\left[\exp\left(c_2/\lambda T\right)-1\right]}\,d\lambda.$$

  - **This integral is (see exercises)**

$$F_{BB} = \sigma T^4$$

**where $\sigma$ is the Stefan-Boltzmann constant, and is $5.67\times10^{-8}$ W-m$^{-2}$-K$^{-4}$.**

**ABSORPTIVITY AND EMISSIVITY**

- **Non-blackbodies emit less radiation than do blackbodies.**
- **The ratio of emitted monochromatic flux to blackbody monochromatic flux is known as the *monochromatic emissivity*, $\varepsilon_\lambda$**

$$\varepsilon_\lambda \equiv F_\lambda / F_{BB\lambda}$$

  - **Emissivity is always between 0 and 1.**
- **The ratio of total emitted flux to blackbody total flux is known as the *gray-body emissivity*, $\varepsilon$**

$$\varepsilon \equiv F/F_{BB}.$$

- **The flux emitted from a gray body is**

$$F = \varepsilon F_{BB} = \varepsilon\sigma T^4 \, .$$

- **Non-blackbodies absorb less radiation than do blackbodies.**
- **The ratio of absorbed monochromatic flux to blackbody absorbed monochromatic flux is known as *absorptivity* ($a_\lambda$).**
    - **Absorptivity is always between 0 and 1.**
- **The ratio of emitted flux to blackbody absorbed flux is known as the *gray-body absorptivity* ($a$).**

**KIRCHOFF'S LAW**

- **Kirchoff's law states that an objects emissivity equals its absorptivity.**

$$a_\lambda = \varepsilon_\lambda \, .$$

    - **Objects that are good emitters at a particular wavelength are good absorbers at that wavelength.**

**EXERCISES**

**1. Starting with**

$$F_{BB\lambda} = \frac{c_1}{\lambda^5\left[\exp\left(c_2/\lambda T\right)-1\right]}$$

**make the variable substitution**

$$x = \frac{c_2}{\lambda T}.$$

**Then find the value of *x* that maximizes $F_{BBx}$ to show that**

$$\lambda_{\max} = \frac{C}{T}$$

**where *C* = 2897 $\mu$m-K.**

**2. Starting with**

$$F_{BB} = \int_0^\infty \frac{c_1}{\lambda^5\left[\exp\left(c_2/\lambda T\right)-1\right]}\, d\lambda$$

**make the variable substitution**

$$x = \frac{c_2}{\lambda T}$$

**to get**

$$F = \frac{c_1 T^4}{c_2^4}\int_0^\infty \frac{x^3}{\exp x - 1}\, dx\,,$$

**and then evaluate the integral to obtain the Stefan-Boltzmann Law.**

**3. What is the blackbody flux of the Sun's surface (*T* = 6000K)?**

**4. What is the blackbody flux of the Earth's surface (*T* = 288K)?**

**5. Are the gray-body and monochromatic emissivities related via** $\varepsilon = \int_0^\infty \varepsilon_\lambda \, d\lambda$ **?**