ESCI 340 – Physical Meteorology
Answers to Selected Exercises for Radiation Lesson 2

1. Starting with Planck's law, make the variable substitution

$$x = \frac{c_2}{\lambda T}.$$

Then find the value of x that maximizes $F_{BB\lambda}$ to show that

$$\lambda_{\max} = \frac{C}{T}$$

where C = 2897 μm-K.

Answer: After the variable substitution we get

$$F_{BBx} = \frac{c_1 T^5}{c_2^5} \frac{x^5}{\exp x - 1}.$$

Taking

$$\frac{\partial F_{BBx}}{\partial x} = 0,$$

gives

$$5 - x = 5/\exp x,$$

which can be solved iteratively to get x = 4.96511. Putting this back into the variable substitution and solving for λ gives Wien's Law.

2. Starting with

$$F_{BB} = \int_0^\infty \frac{c_1}{\lambda^5 \left[\exp(c_2/\lambda T) - 1\right]} d\lambda$$

make the variable substitution

$$x = \frac{c_2}{\lambda T}$$

to get

$$F = \frac{c_1 T^4}{c_2^4} \int_0^\infty \frac{x^3}{\exp x - 1} dx,$$

and then evaluate the integral to obtain the Stefan-Boltzmann Law.

Answer: Using the Schaum's outline or other integral table (in my Schaum's it is eqn. 18.80) you can find that

$$\int_0^\infty \frac{x^3}{\exp x - 1} dx = (3!)\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots\right) = 6 \cdot \frac{\pi^4}{90} = \frac{\pi^4}{15}$$

3. What is the blackbody flux of the Sun's surface (T = 6000K)?

Answer: 7.35×10^7 W/m^2

4. What is the blackbody flux of the Earth's surface (T = 288K)?

Answer: 390 W/m^2