

ESCI 340 – Physical Meteorology
Answers to Selected Exercises for Radiation Lesson 1

1. Show that the solid angle that the disk of a planet makes with a satellite orbiting at a height h above the planet's surface is

$$\omega = 2\pi \left[1 - \frac{\sqrt{2rh + h^2}}{r + h} \right]$$

where r is the radius of the planet.

Answer: We know that

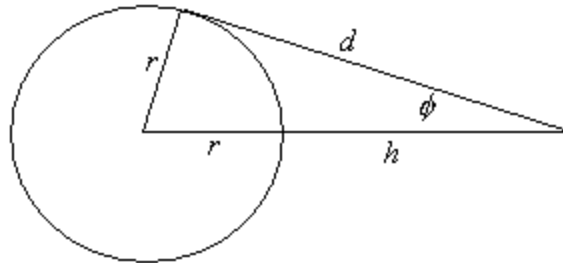
$$\omega = \int d\omega = \int_0^{2\pi} \int_0^{\phi} \sin \phi \, d\phi \, d\theta = 2\pi \int_0^{\phi} \sin \phi \, d\phi = 2\pi(1 - \cos \phi).$$

Referring to the diagram below we see that

$$\cos \phi = \frac{d}{r + h} = \frac{\sqrt{2rh + h^2}}{r + h}$$

so therefore

$$\omega = 2\pi \left[1 - \frac{\sqrt{2rh + h^2}}{r + h} \right].$$



2. What is the solid angle that the disk of the Earth makes with the center of the Sun ($r = 6400$ km, $h = 1.5 \times 10^8$ km)?

Answer: 5.7×10^{-9} sr

3. What is the solid angle that the disk of Jupiter makes with the center of the Sun ($r = 71,000$ km, $h = 7.8 \times 10^8$ km)?

Answer: 2.6×10^{-8} sr

4. What is the solid angle that the moon's disk makes with the center of the Earth? The moon has a radius of 1738 km, and is about 384,400 km from the center of the Earth.

Answer: 6.4×10^{-5} sr

5. If radiation strikes a plane surface with the same intensity (I) regardless of the direction (this is known as *isotropic* radiation), show that the flux is given by

$$F = \pi I .$$

You may use an integral table to solve the integral.

Answer:

$$\begin{aligned} F &= \int_0^{2\pi} \int_0^{\pi/2} I \cos \phi \sin \phi \, d\phi \, d\theta = I \int_0^{2\pi} \int_0^{\pi/2} \cos \phi \sin \phi \, d\phi \, d\theta = 2\pi I \int_0^{\pi/2} \cos \phi \sin \phi \, d\phi \, d\theta \\ &= 2\pi I \left. \frac{\sin^2 \phi}{2} \right|_0^{\pi/2} = \pi I \end{aligned}$$

7. The Sun emits a total of about 3.90×10^{26} W of radiative power. What is the total amount of power that is intercepted (not necessarily absorbed) by the Earth? What is the total flux at the top of the Earth's atmosphere (assume a distance of 1.5×10^8 km from the center of the sun)?

Answer: Total power intercepted by Earth: 1.77×10^{17} W
Total flux: 1370 W/m^2

8. Repeat question 7 for Jupiter.

Answer: Total power intercepted by Jupiter: 8.07×10^{17} W
Total flux: 51 W/m^2

10. A 1000-Watt beam of light is spread uniformly over a narrow cone of angle 2° .

- a. What is the intensity of the beam? (Assume the source is a disk with a radius of 1.5 cm.)

Answer:

$$\text{Use } I = \frac{P_0}{\omega_0 A} \text{ to get } I = 1.47 \times 10^9 \text{ W m}^{-2} \text{ sr}^{-1}$$

- b. The beam shines onto a wall that is perpendicular to the beam, and is 3 meters away from the source. What is the flux on the illuminated portion of the wall?

Answer:

$$\text{Use } F = \frac{P_0}{\omega_0 r^2} \text{ to get } F = 1.16 \times 10^5 \text{ W m}^{-2}$$

- c. The wall is moved to a distance of 6 meters from the source. Now what is the flux on the illuminated portion of the wall?

Answer: $F = 2.89 \times 10^4 \text{ W m}^{-2}$

- d. The wall remains at a distance of 6 meters, but is rotated so that it is now oriented at 45° to the beam. What is the flux on the wall?

Answer: $F = 2.04 \times 10^4 \text{ W m}^{-2}$ (this is answer from c. multiplied by the cosine of the zenith angle.)