ESCI 340 - Cloud Physics and Precipitation Processes
Lesson 10 - Weather Radar
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References:

A Short Course in Cloud Physics, 3rd ed., Rogers and Yau, Ch. 11

Radar Principles

- The components of a radar system are:

  Transmitter: The transmitter generates pulses of radio frequency (RF) radiation.

  Receiver: The receiver detects the returned energy.

  Antenna: The antenna focuses the outgoing beam and the incoming radiation.

- The radar measures the distance to the target by measuring the time between the transmission of a pulse and the detection of the return pulse.
- The total distance traveled by the pulse is $2r$, where r is the distance to the target.
- The time of travel is the total distance divided by the speed of the radiation, c,

$$t = 2r/c. \tag{1}$$

  Therefore, if the travel time is known the distance to the target can be deduced by

$$r = ct/2. \tag{2}$$

Radar Parameters

Pulse Width: Denoted as τ, the pulse width is the transmission duration of the pulse (usually measured in microseconds). Also called *pulse duration.*

- The pulse width determines the range resolution of the radar. If two targets are separated by distance Δr that is less than $c\tau$, they will appear as a single target.
- The pulse width also determines the minimum range of the radar. Since the radar cannot detect incoming pulses while it is transmitting, the minimum range of a radar is

$$r_{min} = c\tau/2. \tag{3}$$

Pulse Repetition Frequency: Abbreviated as *PRF*, and denoted as f_r, this is the number of pulses per second transmitted by the radar.

- The maximum unambiguous range of a radar is

$$r_{max} = \frac{c}{2f_r}. \tag{4}$$

Peak Power: The maximum power of the pulse, denoted as P_t. Usually measured in Watts.

Wavelength: Denoted by λ, this is the wavelength of the radiofrequency wave transmitted by the radar. For weather radars this is in the microwave part of the spectrum. Wavelengths of 3-10 cm are commonly used.

Beamwidth: Denoted as θ, this is the angular width of the radar beam.

Antenna Area: Denoted as A_e, this is the area of the aperture of the antenna.

- For a given beamwidth, as the wavelength increases a larger antenna area is required. Thus, a radar operating at a wavelength of 10 cm will have a larger antenna than one operating at 3 cm.

Antenna Gain: The gain, G, describes the focusing of the radar beam.

- If the antenna transmitted isotropically (evenly in all directions of a sphere) the power incident on a small target of cross-section A_t at a distance r would be

  $$P_{iso} = P_t \frac{\omega}{4\pi}, \tag{5}$$

  where ω is the *solid angle* of the target with respect to the radar.[1]
- If the target is small, such that we can use the small-angle approximation, then the solid angle of the target is $\omega = A_t/r^2$, and the power incident on the target from (5) is

  $$P_{iso} = \frac{P_t A_t}{4\pi r^2}. \tag{6}$$
- The gain is the power intercepted by the target divided by what the power would be if the antenna transmitted isotropically,

  $$G = P_\sigma / P_{iso}. \tag{7}$$
- Therefore, the power intercepted by a target is

  $$P_\sigma = G \frac{P_t A_t}{4\pi r^2}. \tag{8}$$
- The gain is a dimensionless number that is greater than one.
- Gain is related to the wavelength of the radar, and the antenna aperture, by

  $$G = \frac{4\pi A_e}{\lambda^2}. \tag{9}$$

[1] Solid angle is defined as an area on the surface of a sphere divided by the square of the radius. An entire sphere has a solid angle of 4π. For small areas on the sphere we can use the *flat* or *cross-sectional* area rather than the spherical area.

The Radar Equation for a Single Target

- A target of cross-sectional area A_t at a distance r from the radar intercepts an amount of power given by (8), repeated here,

$$P_\sigma = G\frac{P_t A_t}{4\pi r^2}.$$

- If the target reflects (or reradiates) this same amount of energy isotropically, the amount of power received by the radar would be

$$P_r = P_\sigma \frac{\omega_e}{4\pi}, \tag{10}$$

where ω_e is the solid angle of the antenna with respect to the target.

- ω_e is given by

$$\omega_e = \frac{A_e}{r^2}, \tag{11}$$

and using (11) and (8) in (10) results in

$$P_r = P_t \frac{G A_t A_e}{16\pi^2 r^4}. \tag{12}$$

- From (9) we know

$$A_e = \frac{G\lambda^2}{4\pi},$$

and substituting this into (12) results in the radar equation for an isotropically reflecting target of cross-sectional area A_t,

$$P_r = P_t \frac{G^2\lambda^2}{64\pi^3 r^4} A_t. \tag{13}$$

- The cross-sectional area A_t in (13) is usually replaced with the ***backscattering cross section***, which is also called the ***effective cross section***, or ***radar cross section***, σ.
  - The effective cross section is the cross section that the target *appears* to have, and may either be larger or smaller than the actual physical cross section of the target.
  - A target that is very absorptive will reflect and return a small amount of the radar power that hits the target. Thus, the target will appear to be much smaller to the radar, and therefore have a smaller radar cross section. This is the basis of stealth technology to hide aircraft and ships from radar detection.
  - The effective cross section takes into account the shape and composition of the target.

- In terms of effective cross section the radar equation becomes

$$P_r = P_t \frac{G^2\lambda^2}{64\pi^3 r^4}\sigma. \tag{14}$$

Radar Equation for Hydrometeors

- Rain drops are much, much smaller than the wavelength of the radar radiation, and therefore scatter like Rayleigh scatterers (remember why the sky is blue?).
- Rayleigh scattering theory gives the backscattering cross section of a spherical droplet of diameter D as
$$\sigma = \frac{\pi^5}{\lambda^4}|K|^2 D^6, \tag{15}$$
where K is the complex index of refraction.
  - Substituting (15) into (14) gives
$$P_r = P_t \frac{G^2}{64r^4}\frac{\pi^2}{\lambda^2}|K|^2 D^6. \tag{16}$$
- If the radar signal scatters off of multiple droplets within the volume of the radar pulse, the returned power is the sum of the powers from all the droplets,
$$\overline{P}_r = P_t \frac{G^2}{64r^4}\frac{\pi^2}{\lambda^2}|K|^2 \sum D^6. \tag{17}$$
  - We have included an overbar over the P_r to indicate we are really dealing with an average return power.
- Radar ***reflectivity*** is defined as
$$Z = \frac{\sum D^6}{V}, \tag{18}$$
where V is the volume sampled by the radar pulse.
  - Reflectivity has MKS units of m^3, but is usually reported in units of $mm^6\ m^{-3}$.
  - Reflectivity is essentially backscattering cross section per volume.
- The geometric volume sampled by the radar beam is
$$V = \pi\left(\frac{r\theta}{2}\right)^2 \frac{c\tau}{2}. \tag{19}$$
However, the radar beam is not not really uniform over this volume, and the volume doesn't really have 'sharp' edges or boundaries. The radar energy is strongest in the center of the volume, and tapers of toward the edges. Therefore, it is more appropriate to use an ***effective radar volume*** given by
$$V_{eff} = \frac{\pi}{2\ln 2}\left(\frac{r\theta}{2}\right)^2 \frac{c\tau}{2}. \tag{20}$$

- Using (20) in (18), and then rearranging, gives us

$$\sum D^6 = \frac{\pi}{16 \ln 2}(r\theta)^2 c\tau Z, \tag{21}$$

and using (21) in (17) result in the ***radar equation for hydrometeors***,

$$\overline{P}_r = \frac{\pi^3 c}{1024 \ln 2} \underbrace{\left(\frac{P_t \tau G^2 \theta^2}{\lambda^2}\right)}_{\text{Term A}} \underbrace{\left(|K|^2 \frac{Z}{r^2}\right)}_{\text{Term B}}. \tag{22}$$

  - Term A consists of properties of the radar.
  - Term B consists of properties of the target.

Reflectivity and Rainfall Rate

- The definition of reflectivity given in (18) is for a discrete raindrop distribution. For a continuous distribution of raindrops the reflectivity is

$$Z = \int_0^\infty D^6 n_d(D) dD. \tag{23}$$

- Using the Marshall-Palmer distribution in (23) yields

$$Z = \frac{720 n_0}{\Lambda^7}, \tag{24}$$

or

$$\Lambda = \left(\frac{720 n_0}{Z}\right)^{1/7}. \tag{25}$$

- In a prior lesson we found that the slope factor of the Marshall-Palmer distribution is related to rainfall rate via

$$\Lambda = \left(\frac{\pi n_0 \overline{u}}{R}\right)^{1/4}. \tag{26}$$

- Equating (25) and (26) and solving for Z yields the theoretical Z-R relation

$$Z = \frac{720}{(\pi \overline{u})^{7/4} n_0^{3/4}} R^{7/4}. \tag{27}$$

- In practice Z-R relations are found empirically, with an often used relation of

$$Z = 200 R^{1.6}, \tag{28}$$

where R is in mm hr^{-1}, and Z is in mm^6 m^{-3}.

Decibels of Reflectivity

- Since Z can vary by a wide range it is convenient to use a logarithmic form for reflectivity. We define decibels of reflectivity, or $\boldsymbol{dBZ}$, as

$$dBZ = 10\log_{10}\left(\frac{Z}{\text{mm}^6\ \text{m}^{-3}}\right). \tag{29}$$

- Typical values of dBZ are 50 or greater in the core of a thunderstorm, and around 30 for light rain.

Bright Band

- Snow and frozen precipitation will usually have a smaller value of reflectivity because it is made of ice, which has a lower index of refraction than liquid water.
- When frozen precipitation falls to a level where the temperature is above freezing it begins to melt.
- The water surface on the outside will cause an increase in reflectivity, which results in a bright band on the radar image at the height near the freezing level.
- The bright band does not extend to the surface because once the snowflake completely melts it falls faster. Therefore, the droplet concentration decreases as altitude decreases, resulting in lower reflectivity.

Radar Beam Height and Elevation Angle

- As the radar beam gets further from the radar the altitude of the beam above the ground gets larger. This is due to two effects:
  - Curvature of the Earth.
  - Refraction of the beam upwards.
- The altitude of the beam $h(r)$ with distance from the radar r is given by

$$h(r) = h_0 - ka + \sqrt{r^2 + k^2a^2 + 2rka\sin\varphi}, \tag{30}$$

  where φ is the elevation angle, k is a standard refraction coefficient, a is the radius of the Earth, and h_0 is the height of the radar antenna.
- Figure 1 shows beam height versus altitude for various elevation angles.
- Because the beam rises off the ground with distance, shallow precipitation at large distances may be completely missed by the radar.
- The lowest elevation angle is usually 0.5°.

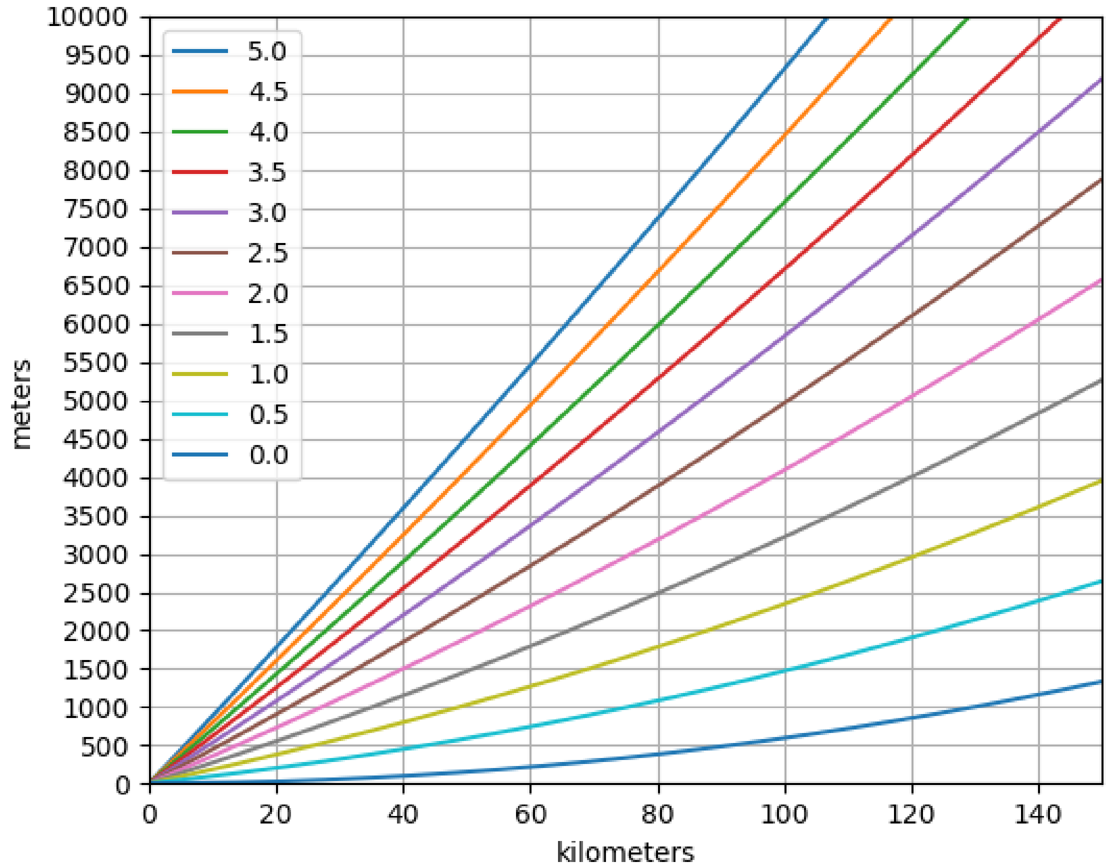

Figure 1: Beam height versus range for various elevation angles. Values used in (30) are $h_0 = 10$ m and $k = 4/3$.

Exercises

1. Show that for the Marshall-Palmer drop-size distribution

$$Z = \frac{720 n_0}{\Lambda^7}.$$

2. (a) Show that for the Marshall-Palmer drop-size distribution

$$Z = \frac{720}{(\pi \overline{u})^{7/4} n_0^{3/4}} R^{7/4}.$$

   (b) Use the formula above to find Z and dBZ for a rainfall rate of 1 inch per hour. Use $n_0 = 0.08$ cm^{-1} and $\overline{u} = 5$ m s^{-1}. Be careful with your units!

(c) An often used empirical Z-R relation is

$$Z = 200R^{1.6}.$$

Use this formula to find Z and dbZ for a rainfall rate of 1 inch per hour.

(d) Why do you think the empirical formula gives a different answer than the theoretical formula?