ESCI 340 - Cloud Physics and Precipitation Processes
Lesson 4 - Convection
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References:

Glossary of Meteorology, 2nd ed., American Meteorological Society
A Short Course in Cloud Physics, 3rd ed., Rogers and Yau, Ch. 4

Adiabatic Mixing of Air Parcels

- If two air parcels are adiabatically mixed together, many thermodynamics properties of the mixture are a ***mass-weighted mean*** of their properties before mixing.
  - A mass-weighted mean of some property s of two air parcels of masses m_1 and m_2 is given by the formula

$$\overline{s}_m = \frac{m_1}{m_1 + m_2} s_1 + \frac{m_2}{m_1 + m_2} s_2. \tag{1}$$

- Formula (1) applies exactly if s is specific humidity q, and approximately for mixing ratio r and potential temperature θ.
- If the air parcels being mixed are also at the same pressure (***isobaric mixing***), then temperature and vapor pressure also mix as mass-weighted means, and (1) also applies.
- Adiabatic mixing of two initially unsaturated air parcels may actually result in a saturated air parcel.
  - This is why we can sometimes 'see our breath' on cold days.
- The concept of mass-weighted mean can be applied to a continuous layer of air as follows:
  - We imagine the layer consisting of a series of N very thin air parcels, each having a horizontal area A and thickness Δz_i.
  - The mass-weighted mean is given by the sum

$$\overline{s}_m = \frac{\sum_i m_i s_i}{\sum_i m_i}. \tag{2}$$

  - Each parcel has a mass given by $\rho_i A \Delta z_i$, so that (2) becomes

$$\overline{s}_m = \frac{\sum_i \rho_i A \Delta z_i s_i}{\sum_i \rho_i A \Delta z_i} = \frac{\sum_i \rho_i \Delta z_i s_i}{\sum_i \rho_i \Delta z_i}. \tag{3}$$

- In the limit as the thicknesses of the air parcels go to zero the summation turns into an integral, and the formula for the mass-weighted mean of a layer becomes

$$\overline{s}_m = \frac{\int_{z_1}^{z_2} \rho s dz}{\int_{z_1}^{z_2} \rho dz}. \tag{4}$$

- Formula (4) applies only to those parameters s that do not change as the air parcel moves up or down.
  - Thus, it can be applied to specific humidity, mixing ratio, and potential temperature.
  - Formula (4) <u>*cannot*</u> be applied to temperature.
- Formula (4) can be written as a derivative with respect to pressure. The hydrostatic equation allows us to write $\rho dz = -gdp$, so that (4) becomes[1]

$$\overline{s}_m = \frac{\int_{p_2}^{p_1} gsdp}{\int_{p_2}^{p_1} gdp} = \frac{1}{\Delta p}\int_{p_2}^{p_1} sdp, \tag{5}$$

  where $\Delta p = p_1 - p_2$.
- During adiabatic mixing the specific humidity and potential temperature mix as mass-weighted means, and take on constant values in the mixed layer. The mixing ratio also mixes to a close approximation as a mass-weighted mean From (5) these values are

$$\overline{q}_m = \frac{1}{\Delta p}\int_{p_2}^{p_1} qdp \tag{6}$$

$$\overline{r}_m = \frac{1}{\Delta p}\int_{p_2}^{p_1} rdp \tag{7}$$

$$\overline{\theta}_m = \frac{1}{\Delta p}\int_{p_2}^{p_1} \theta dp. \tag{8}$$

- On a diagram using a logarithmic pressure axis the averages for (6), (7), and (8) can be approximated graphically, by using the ***method of equal areas***.
  - The mass-weighted mean mixing ratio, r_m, is the isohume that splits the dewpoint profile into two equal areas, as shown in Figure 1.

[1] We can usually ignore, without significant error, the fact that g decreases with altitude. This allows it to be removed from the integrals.

- The mass-weighted mean potential temperature, θ_m, is the adiabat that splits the temperature profile into two equal areas, as shown in Figure 1

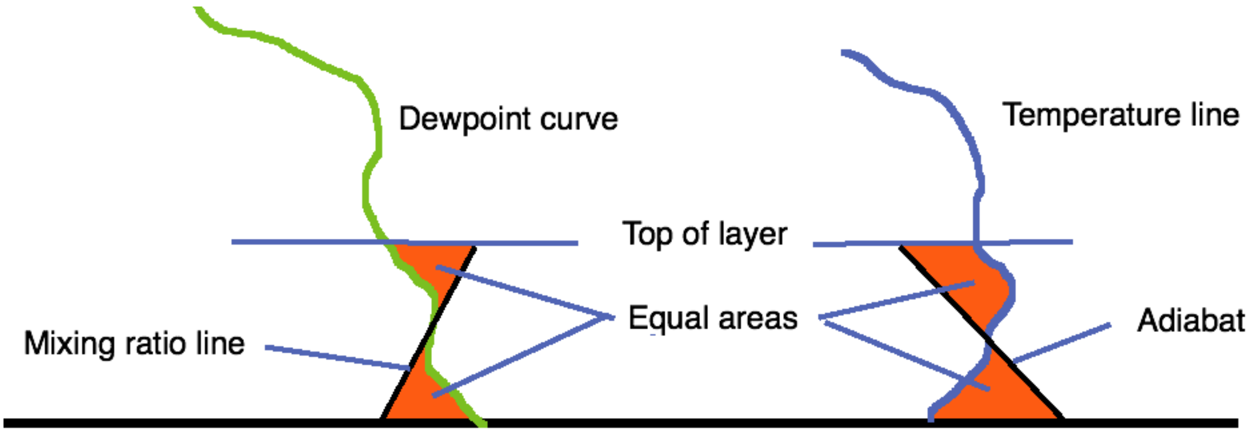

Figure 1: Equal-area method for determining mass-weighted mean of mixing ratio and potential temperature.

- Adiabatic mixing is why surface temperatures on a windy night will be warmer than on a calm night.

Convective Mixing and the CCL

- Another important mixing process for the atmosphere is ***convective mixing***, which occurs when a layer of air is heated from below and the upward and downward overturning of the layer mixes the air parcels.
- Convective mixing is ***not adiabatic***.
  - In a convectively mixed layer θ still takes on a constant value, but it is ***not*** given by (8).
  - Instead, θ in a convectively mixed layer will be the value of the adiabat through the surface temperature.
  - Mixing ratio in a convectively mixed layer will still be given by (7).
- A morning sounding usually has a surface inversion.
- As the solar heating warms the ground a convectively mixed layer begins to form, eating away at the surface inversion.
  - A parcel at the surface can rise up the dry adiabat until it reaches the top of the convectively-mixed layer.
  - As the parcel rises it conserves it mixing ratio.

- If the parcel reaches an altitude where the saturation mixing ratio is equal to the parcels mixing ratio, a cloud will form at this altitude. This is known as the ***convective condensation level***, or ***CCL***.

- On a skew-T diagram the CCL is found by following the isohume through the surface dewpoint up to where it crosses the temperature profile.
  - This assumes that the parcel does not mix with it's environment during ascent. Therefore, the CCL found from this procedure is called the ***parcel method*** CCL.

- A more accurate assumption is that the parcel will mix and become diluted as it ascends. Therefore, a better approximation to the CCL is found by averaging the mixing ratio through a layer of air near the surface, and following this average mixing ratio line up to the temperature sounding.
  - The CCL found from averaging the mixing ratio in a layer of air is known as the ***mixing method*** CCL.

- Figure 2 illustrates the locations of the CCL for both the mixing and the parcel methods.

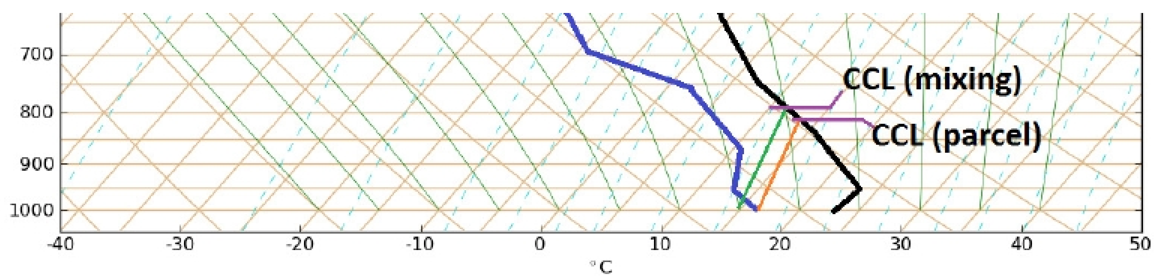

Figure 2: Example Skew-T showing location of convective condensation level (CCL) using both the parcel and the mixing methods.

- The ***convective temperature***, T_c, is the temperature that the surface must reach in order for an air parcel to ascend dry adiabatically to reach the CCL.
  - The convective temperature is found by following a dry adiabat downward from the CCL to the surface, and reading the temperature at that point.
  - Figure 3 illustrates the determination of the convective temperature.

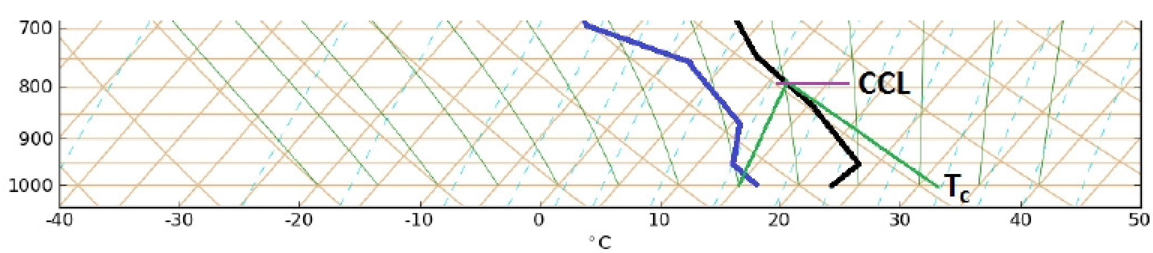

Figure 3: Example Skew-T showing how to find the convective temperature, T_c.

Lifted Ascent

- For air parcels lifted *mechanically* rather than convectively, the cloud bases will be at the ***lifting condensation level***, or ***LCL***.
  - The LCL is located by following the mixing ratio contour through the surface dewpoint and finding where it crosses the adiabat through the surface temperature (see Fig. 4).
- If the parcel is lifted beyond the LCL it will follow a moist adiabat.
  - If the moist adiabat eventually crosses the environmental sounding, then the parcel will be warmer than it's environment and will continue to rise on it's own. The level at which this occurs is known as the ***level of free convection***, or ***LFC*** (see Fig. 4).
  - Not all soundings will have an LFC.

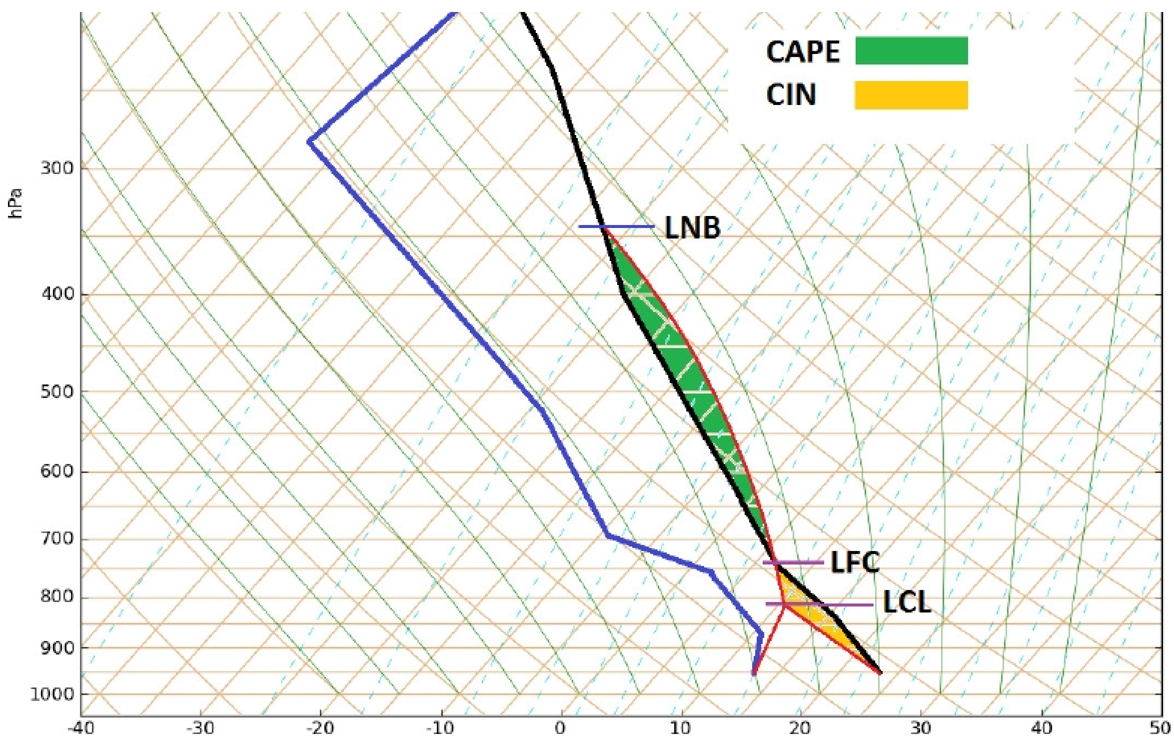

Figure 4: Example Skew-T diagram showing the lifting condensation level (LCL), level of free convection (LFC), level of neutral buoyancy (LNB), and areas of convective available potential energy (CAPE) and convective inhibition (CIN).

- Once a parcel is beyond the LFC it will continue to ascend along a moist adiabat until it once again intersects the environmental sounding. Beyond this point the parcel will be cooler than the surrounding air, and will have negative buoyancy. The level at which this occurs is known as the ***level of neutral buoyancy***, or ***LNB***.[2]

Convective Available Potential Energy

- In the previous lesson we established that the vertical acceleration on an air parcel is given by
$$a'_z = g\frac{T' - T}{T}. \tag{9}$$
- We also know that the acceleration can be written as $a_z = dU/dt$ where U is the vertical velocity of the air parcel.[3] From the chain rule we can write
$$a'_z = \frac{dU}{dt} = \frac{dU}{dz}\frac{dz}{dt} = \frac{dU}{dz}U = \frac{1}{2}\frac{dU^2}{dz}. \tag{10}$$
- Combining (9) and (10) and rearranging results in
$$dU^2 = 2g\left(\frac{T' - T}{T}\right)dz. \tag{11}$$
- We can use (11) to calculate the vertical velocity of the parcel once it reaches the LNB. We do this by integrating (11) from the LFC to the LNB we get
$$U^2_{LNB} - U^2_{LFC} = 2g\int_{z_{\mathrm{LFC}}}^{z_{\mathrm{LNB}}}\left(\frac{T' - T}{T}\right)dz, \tag{12}$$
and if we assume that the parcel's velocity at the LFC is zero, this becomes
$$U^2_{LNB} = 2g\int_{z_{\mathrm{LFC}}}^{z_{\mathrm{LNB}}}\left(\frac{T' - T}{T}\right)dz. \tag{13}$$
- Equation (12) has another useful interpretation. Recognizing that kinetic energy per unit mass is
$$E = \frac{U^2}{2},$$
then (12) becomes
$$E_{LNB} - E_{LFC} = g\int_{z_{\mathrm{LFC}}}^{z_{\mathrm{LNB}}}\left(\frac{T' - T}{T}\right)dz. \tag{14}$$

[2] The level of neutral buoyancy is also called the *equilibrium level*, or *EL*.

[3] Normally meteorologists use w to indicate vertical velocity, but in order to be consistent with Rogers and Yau I will use U for the vertical velocity of the air parcel. The prime on U is not needed, since the only vertical velocity we are dealing with is that of the air parcel.

- The difference in kinetic energy per mass between the LNB and LFC, $E_{LNB} - E_{LFC}$, is called the ***convective available potential energy***, or ***CAPE***, and so we have

$$\mathrm{CAPE} = g \int_{z_{\mathrm{LFC}}}^{z_{\mathrm{LNB}}} \left(\frac{T' - T}{T} \right) dz. \tag{15}$$

  - The units for CAPE are energy per mass, J kg^{-1}.

- The equation for CAPE can be converted to pressure coordinates (see Exercises), and is

$$\mathrm{CAPE} = -R_d \int_{p_{\mathrm{LFC}}}^{p_{\mathrm{LNB}}} (T' - T)\, d\ln p. \tag{16}$$

  - On a skew-T diagram T is represented by the temperature sounding and T' is represented by the moist adiabat through the LFC.
  - The right-hand side of (16) is represented by the green shaded area shown in Fig. 4.
  - CAPE is also called the ***positive area*** on the skew-T.

- ***Convective inhibition***, or ***CIN***, is the work required to lift the parcel from the surface to the LFC.

  - The equation for CIN is similar to that for CAPE, only with different limits of integration,

$$\mathrm{CIN} = -R_d \int_{p_0}^{p_{\mathrm{LFC}}} (T' - T)\, d\ln p, \tag{17}$$

    where p_0 is the surface pressure.
  - CIN is represented by the orange shaded area on the skew-T diagram in Fig. 4.
  - CIN is also called the ***negative area*** on the skew-T.

- To account for humidity's effects on buoyancy the virtual temperature, T_v should really be used when calculating CAPE and CIN.

  - When virtual temperature is used in the calculations the CAPE is often denoted as CAPEV or CAPE$_v$, and CIN is denoted as CINV or CIN$_v$.
  - The equations for these quantities are

$$\mathrm{CAPE}_v = -R_d \int_{p_{\mathrm{LFC}}}^{p_{\mathrm{LNB}}} (T_v' - T_v)\, d\ln p \tag{18}$$

$$\mathrm{CIN}_v = -R_d \int_{p_0}^{p_{\mathrm{LFC}}} (T_v' - T_v)\, d\ln p. \tag{19}$$

Forecasting CAPE

- CAPE is often computed from the morning sounding. However, convection doesn't usually fire up until the afternoon. So, the morning value of CAPE may not be particularly relevant for the afternoon's thunderstorms.

- The Storm Prediction Center (SPC) computes a forecast CAPE value by making assumptions about what the temperature and dewpoint of the surface parcel will be in the afternoon.

- Their assumptions are:[4]

  - The forecast surface temperature is the temperature that the parcel at 850 mb would have if it were brought dry-adiabatically to the surface, plus 2°C.
  - The forecast surface mixing ratio is the mean mixing ratio of the lowest 100 mb of the sounding.

- The LCL and CAPE are then computed based on this forecast surface parcel.

Limitations of CAPE

- From (13) and (15) we see that the maximum updraft speed is related to CAPE via

$$U_{max} = U_{\mathbf{LNB}} = \sqrt{2 \times \text{CAPE}}. \tag{20}$$

- Equation (20) will overestimate the maximum updraft speed because of several factors.

Water burden: The expressions for CAPE assumed that any liquid water immediately falls from the air parcel. In reality there is a significant amount of liquid water that remains with the parcel, which adds to the mass of the parcel, and therefore slows it down.

Compensating subsidence: In areas of convection the regions between updrafts are occupied by downward motion, or subsidence. This warms the air outside of the updraft, and decreases the temperature difference between the parcel and the environment, reducing the buoyancy.

Entrainment: Along the edges of the convective updraft there will be mixing of drier and cooler air into the buoyant plume. This cools the air within the updraft through both evaporative cooling and mixing, and reduces the buoyancy.

Aerodynamic resistance: The rising plume is slowed down by having to push air out of its way.

[4] http://www.spc.noaa.gov/exper/soundings/help/index.html

Exercises

1. Derive (16) from (15). Hint: Use the hydrostatic equation to substitute for dz. Then use the ideal gas law to substitute for ρ.