

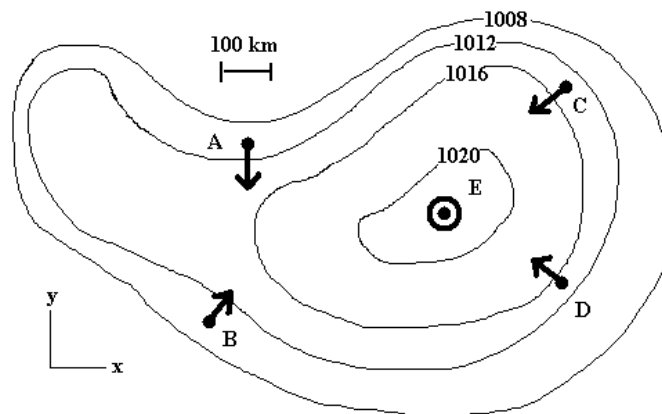
Introductory Meteorology for Scientists  
Lesson 11 – Wind  
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### COORDINATES AND VELOCITY

- In meteorology we use the following coordinate system:
  - The  $x$ -coordinate increases eastward
  - The  $y$ -coordinate increases northward
  - The  $z$ -coordinate increases upward
- The velocity components along each coordinate direction are defined as
  - $u \equiv dx/dt$  ;  $u$  is the speed in the eastward direction (*zonal velocity*)
  - $v \equiv dy/dt$  ;  $v$  is the speed in the northward direction (*meridional velocity*)
  - $w \equiv dz/dt$  ;  $w$  is the speed in the upward direction (*vertical velocity*)

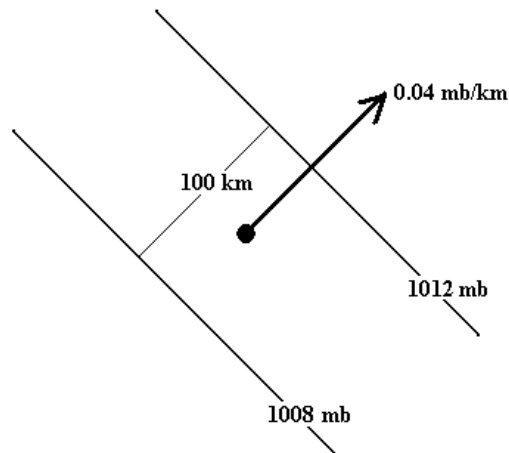
### GRADIENT

- The gradient of a scalar is a vector that points in direction of steepest increase of the scalar.
- If a contour of the scalar field is plotted (such as isotherms or isobars) the gradient at a given point is a vector that is oriented at  $90^\circ$  to the contours and pointed toward higher values.
  - The example below shows the direction of the pressure gradient at several points.



- The magnitude of the gradient at a point on a contour map can be found by dividing the contour interval by the shortest distance between contours across

the point. In the example below the magnitude and direction of the pressure gradient is shown.



- The magnitude was found by

$$|\nabla p| \cong \left| \frac{\Delta p}{\Delta n} \right| = \frac{1012mb - 1008mb}{100km} = 0.04mb / km$$

- The magnitude of the gradient increases if the contours become closer together, and decreases as they get farther apart.
- When the contours are packed closely together it is often referred to as a *tight gradient*.
- When the contours are far apart it is often referred to as a *loose gradient*.

## FORCES AND MOTION

- The velocity of an object is the combination of its speed and the direction in which it is traveling
- Acceleration is defined as the change in velocity. This can be either
  - A change in speed (either increase or decrease)
  - A change in direction
- If the forces acting on an object are not in balance, then the object will accelerate.
- If the forces acting on an object are in balance (or, if there are no forces acting on the object), then the object's velocity will not change.
  - If it were stopped it would stay stopped

- If it were moving, it would continue to move in the same direction and at the same speed.
- An object moving at a uniform speed around a circle is still accelerating, because its velocity is changing (the direction is changing). This acceleration is called the centripetal acceleration. Since it is accelerating, the forces on an object moving in a steady circle are not in balance.
- The wind obeys Newton's second law of motion

$$\sum \vec{F} = m\vec{a}. \quad (1)$$

- Another way of writing Newton's law is

$$\vec{a} = \frac{\sum \vec{F}}{m}. \quad (2)$$

- Acceleration is just force per unit mass.
- Meteorologists are sometimes sloppy, referring to acceleration as force.

#### PRESSURE GRADIENT FORCE (PGF)

- This is the force caused by the difference in air pressure on opposite sides of the air parcel. This force tries to push the parcel toward lower pressure.
- The acceleration due to the pressure-gradient force is

$$\vec{a}_{PGF} = -\frac{1}{\rho} \nabla p \quad (3)$$

(the subscript just shows that this is the acceleration due to the pressure gradient force, and is not necessarily the total (or net) acceleration.

- Notice that the pressure-gradient acceleration is opposite to the pressure gradient itself.
  - *The pressure-gradient acceleration is always perpendicular to the isobars and pointing toward lower pressure.*
- The spacing of the isobars is called the *pressure gradient*. A closer spacing means a *tighter* or *steeper* gradient. A greater spacing means a *looser* gradient.
  - The tighter the pressure gradient, the stronger the pressure gradient force will be.
- On an upper-level map, which is a constant pressure surface, we cannot use (3) for the pressure gradient acceleration. Instead we use

$$\vec{a}_{PGF} = -g_0 \nabla Z \quad (4)$$

where  $Z$  is the height of the pressure surface and  $g_0$  is the reference value of gravity, which is  $9.80665 \text{ m s}^{-1}$ .

### CORIOLIS FORCE (CF)

- This force is an apparent force that results from using a rotating coordinate system.
- The Coriolis force acts horizontally at  $90^\circ$  to the right or left of motion (depending on which direction the coordinate system is rotating).
  - Note: There is a vertical component to the Coriolis force, but it is very small and can usually be ignored.
- The magnitude Coriolis acceleration is linear in speed, and is given as

$$|\vec{a}_{COR}| = fV \quad (5)$$

where  $f$  is the Coriolis parameter,

$$f = 2\Omega \sin \phi. \quad (6)$$

- $\Omega$  is the angular speed of the Earth, and  $\phi$  is latitude.
- Since the Earth rotates through  $2\pi$  radians in one Sidereal Day (23.93447 hours), the angular speed of the Earth is

$$\Omega = \frac{2\pi}{23.93447 \text{ hr}} \times \frac{\text{hr}}{3600 \text{ s}} = 7.292 \times 10^{-5} \text{ s}^{-1}.$$

- The Coriolis acceleration acts  $90^\circ$  to the right of the wind in the Northern Hemisphere, and  $90^\circ$  to the left of the wind in the Southern Hemisphere.
  - (Note: There actually is a Coriolis acceleration due to the vertical motion also, but it is small in comparison the that due to the horizontal motion. This is discussed in more advance courses.
- In vector form, the Coriolis acceleration is given by

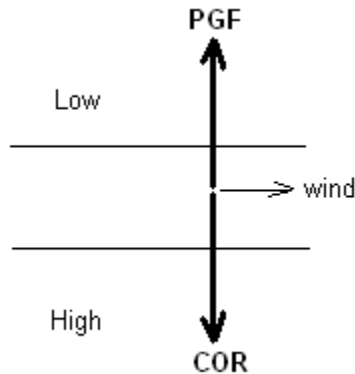
$$\vec{a}_{COR} = -\hat{k} \times f\vec{V}. \quad (7)$$

## GEOSTROPHIC BALANCE

- If the pressure gradient force and the Coriolis force are the only two forces we are concerned with, then the total acceleration of the air is just the sum of the pressure gradient and Coriolis accelerations, (3) and (7),

$$\vec{a} = -\frac{1}{\rho}\nabla p - \hat{k} \times f\vec{V} \quad (8)$$

- If the Coriolis force exactly balances the pressure gradient force then the net acceleration is zero, and we have what is known as *geostrophic balance*, and the wind in such balance is called the *geostrophic wind*.
- The force diagram for geostrophic balance in the Northern Hemisphere is



- For the Southern Hemisphere the force arrows would be identical, but the wind would be in the opposite direction, since Coriolis is to the left in the Southern Hemisphere.
- The magnitude (or speed) of the geostrophic wind is

$$V_g = \frac{1}{\rho f} |\nabla p|. \quad (9)$$

- Properties of the geostrophic wind are:
  - The geostrophic wind blows parallel to the isobars with lower pressure to the left.
  - The greater the pressure gradient, the stronger the geostrophic wind.
  - A given pressure gradient will give a stronger geostrophic wind at lower latitudes (because  $f$  gets smaller at lower latitudes).

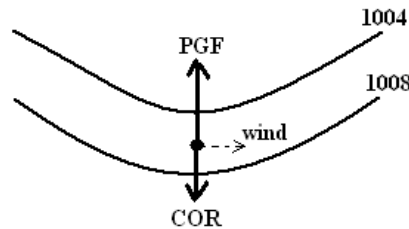
- To a first approximation the winds in the atmosphere can be considered to be in geostrophic balance. Therefore, we can look at a weather map and can judge what the wind flow is from the pattern of the isobars.
  - *BUYS BALLOTT'S LAW* – In the northern hemisphere, if you stand with your back to the wind, low pressure will be to your left.
- The speed of the geostrophic wind can be approximated by

$$V_g \cong \frac{1}{\rho f} \left| \frac{\Delta p}{\Delta n} \right| \quad (10)$$

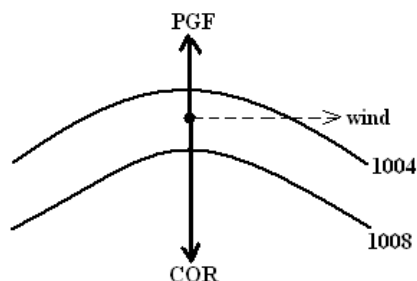
where  $\Delta p$  is the difference in pressure between adjacent isobars (in Pascals, not millibars), and  $\Delta n$  is the horizontal distance between the two adjacent isobars.

### GRADIENT WIND

- If the flow is curved, then the wind is accelerating (since its direction is changing). Therefore, the forces on the air are no longer in balance. Whether the flow is curved to the left or right determines which force is stronger than the other.
- If the flow is curved to the left (cyclonic flow) then the pressure gradient force must be stronger than the Coriolis force.

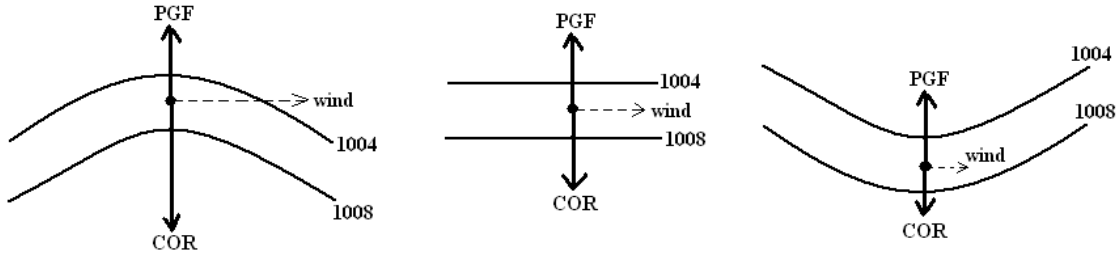


- If the flow is curved to the right (anticyclonic flow) then the pressure gradient force must be weaker than the Coriolis force.



- The geostrophic wind, adjusted for curvature, is known as the *gradient wind*.

- The gradient wind will be weaker than the geostrophic wind (*subgeostrophic*) in cyclonically curved flow, and stronger than the geostrophic wind (*supergeostrophic*) in anticyclonically curved flow.



- If we are dealing with winds well away from the surface of the earth, and flowing in a straight line, then for large-scale motions the winds are usually close to being geostrophic. If friction is present, or if the flow is curved, then geostrophic balance no longer applies.
- The magnitude of the gradient wind can be found by recognizing that in

$$\vec{a} = -\frac{1}{\rho} \nabla p - \hat{k} \times f \vec{V} \quad (11)$$

the acceleration is just the *centripetal acceleration*

$$|\vec{a}| = V^2/R \quad (12)$$

where  $R$  is the radius of curvature.

### GRADIENT WIND AROUND A CYCLONE

- The centripetal acceleration is always toward the center of the curve (the inside of the curve).
- For a cyclone (low pressure) then the pressure gradient acceleration and centripetal acceleration are in the same direction, and the acceleration balance is

$$\frac{V^2}{R} = \frac{1}{\rho} |\nabla p| - fV \quad (13)$$

or

$$V^2 + fRV - \frac{R}{\rho} |\nabla p| = 0 \quad (14)$$

which is a quadratic equation for  $V$  and can be solved to get

$$V = -\frac{fR}{2} \pm \frac{1}{2} \sqrt{f^2 R^2 + \frac{4R}{\rho} |\nabla p|}. \quad (15)$$

- Since  $V$  is a positive value we only take the + root and so for a cyclone

$$V_{grad} = -\frac{fR}{2} + \frac{1}{2} \sqrt{f^2 R^2 + \frac{4R}{\rho} |\nabla p|}. \quad \text{Cyclone} \quad (16)$$

- Since we know that

$$fV_g = \frac{1}{\rho} |\nabla p| \quad (17)$$

we can substitute (17) into (16) and simplify to get the gradient wind in terms of the geostrophic wind,

$$V_{grad} = \frac{fR}{2} \left( \sqrt{1 + 4V_g / fR} - 1 \right). \quad \text{Cyclone} \quad (18)$$

## GRADIENT WIND AROUND AN ANTICYCLONE

- For an anticyclone (high pressure) then the pressure gradient acceleration and centripetal acceleration are in opposite direction, and the acceleration balance is

$$\frac{V^2}{R} = -\frac{1}{\rho} |\nabla p| + fV \quad (19)$$

which when solved for  $V$  gives

$$V_{grad} = \frac{fR}{2} \pm \frac{1}{2} \sqrt{f^2 R^2 - \frac{4R}{\rho} |\nabla p|}. \quad (20)$$

- Equation (20) has two solutions for the wind. One will be small, and one will be large. It is the small value that is encountered in nature, so for an anticyclone we have

$$V_{grad} = \frac{fR}{2} - \frac{1}{2} \sqrt{f^2 R^2 - \frac{4R}{\rho} |\nabla p|}. \quad \text{Anticyclone} \quad (21)$$

- The equation for the anticyclone is interesting because the radical must be a positive value. This actually places a limit on how strong the pressure gradient can be in an anticyclone

$$|\nabla p| < f^2 \rho R / 4. \quad (22)$$



- Equation (22) is the reason that you never see anticyclones with very tight pressure gradients toward the center.
- Equation (21) can also be written in terms of the geostrophic wind as

$$V_{grad} = \frac{fR}{2} \left(1 - \sqrt{1 - 4V_g / fR}\right). \quad \textit{Anticyclone} \quad (23)$$

## THE ROSSBY NUMBER

- The acceleration balance for curved flow is

$$\vec{a} = -\frac{1}{\rho} \nabla p - \hat{k} \times f\vec{V}. \quad (24)$$

The terms in this equation have magnitudes of

$$\frac{V^2}{R} = \frac{1}{\rho} |\nabla p| + fV \quad (25)$$

where the  $\sim$  symbol shows that we are only looking at the orders of magnitude of the terms and not exact values.

- If we divide (25) by  $fV$  we get

$$\frac{V}{fR} = \frac{1}{fV\rho} |\nabla p| + 1. \quad (26)$$

- The combination  $V/fR$  is called the *Rossby number*

$$R_o = V/fR. \quad (27)$$

- If the Rossby number is much, much less than one then the centripetal acceleration is extremely small compared to the Coriolis acceleration, and all we are left with in (25) is a balance between the pressure gradient and the Coriolis, which is *geostrophic* balance.
- If the Rossby number is of the order of one then all three terms in (25) are important and we have *gradient* balance.
- If the Rossby number is much, much greater than one then the Coriolis term is negligible compared to the centripetal acceleration and so the Coriolis can be ignored. This is called *cyclostrophic* balance.
- The term *much, much* implies an order of magnitude (factor of 10 or more) greater. For example, 5 is not much, much greater than 1, but 15 is much, much greater than 1.

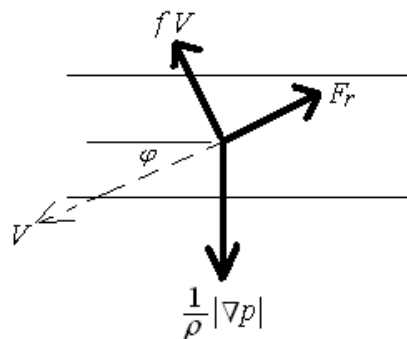
- We use the symbol  $\gg$  for much, much greater, and  $\ll$  for much, much less.
- The table below summarizes the different regimes of the Rossby number.

<u>Regime</u>	<u>Balance</u>	<u>Important Terms</u>
$R_o \ll 1$	Geostrophic	Pressure gradient and Coriolis
$R_o \sim 1$	Gradient	All terms
$R_o \gg 1$	Cyclostrophic	Centripetal acceleration and pressure gradient

- Note that vortexes such as tornadoes, dust devils, or the whirlpool in your toilet or bathtub all have Rossby numbers that are much, much larger than one, and so are in cyclostrophic balance.
- Coriolis does not cause bathtub whirlpools to spin in opposite direction in the Southern Hemisphere compared to the Northern Hemisphere.

## FRICITION

- *Friction (FR)* – Friction always acts to slow the air down, so it is always acting opposite to the direction of motion.
- Friction is only important within a few kilometers of the earth’s surface (in what is known as the *planetary boundary layer*). Above the planetary boundary layer we can assume that there is no friction.
- If friction is present, the wind can no longer be in geostrophic balance.
- Friction results in a slowing of the wind. This causes the wind to blow slightly across the isobars toward lower pressure.
- The force balance for steady flow with friction is shown in the diagram



- In order to balance the friction, Coriolis, and pressure gradient accelerations the following relations must hold,

$$F_r \cos \varphi - fV \sin \varphi = 0 \quad (28)$$

$$F_r \sin \varphi + fV \cos \varphi - \frac{1}{\rho} |\nabla p| = 0. \quad (29)$$

- Friction can sometimes be considered a linear function of velocity so that

$$F_r = C_D V, \quad (30)$$

where  $C_D$  is called the drag coefficient. In this case, the cross-isobar angle calculated from (28) is

$$\tan \varphi = C_D / f. \quad (31)$$

- Where the drag coefficient is larger, the cross-isobar angle is larger.
- For the same drag coefficient, the cross-isobar angle is larger at low latitudes than at higher latitudes.
- Using (30) in (29) allows us to solve for the wind speed with friction, which is

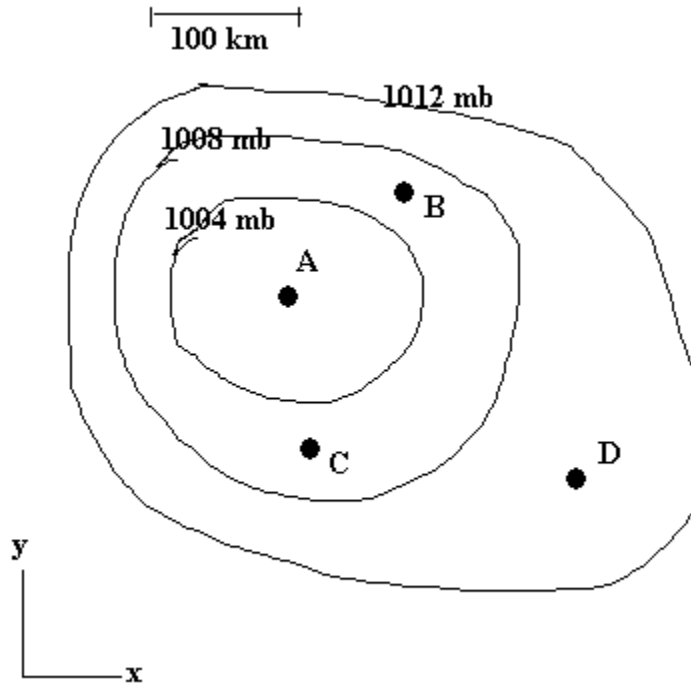
$$V = \frac{|\nabla p|}{\rho(f \cos \varphi + C_D \sin \varphi)} = \frac{fV_g}{f \cos \varphi + C_D \sin \varphi}. \quad (32)$$

## FRICION AND CURVED FLOW TOGETHER

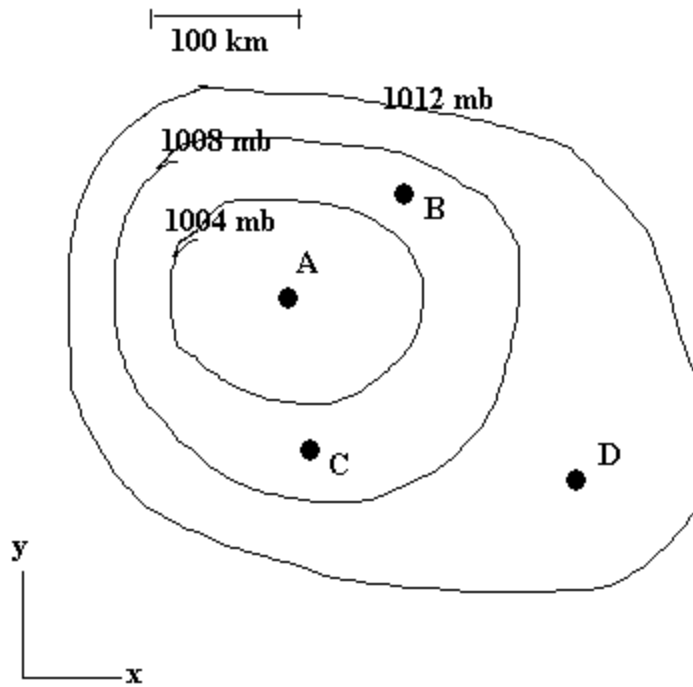
- In a low-pressure system near the surface of the Earth, friction will cause the wind to blow toward the center of the low. This results in convergence within the low, and upward vertical motion.
- In a high-pressure system near the surface of the Earth, friction will cause the wind to blow away from the center of the high. This results in divergence within the high, and downward vertical motion.

## EXERCISES

1. On the picture below, draw an arrow representing the direction of the pressure gradient at each point A, B, C, and D.



2. For the picture above, calculate the magnitude of the pressure gradient (in Pascals per meter) at each point A, B, C, and D.
3. On the picture below (this is the same picture as in problem 1), draw arrows representing the direction of the geostrophic wind at each point A, B, C, and D.
4. Go through all the steps to derive Equation (32).



4. For the picture above calculate the magnitude of the geostrophic wind at each point A, B, C, and D. Use  $\rho = 1.23 \text{ kg/m}^3$ , and  $f = 10^{-4} \text{ s}^{-1}$ .
5. For the picture above, fill in the table below telling whether the  $x$  and  $y$  components of the geostrophic wind are positive or negative.

Point	$u_g$	$v_g$
A		
B		
C		
D		