

ESCI 241 – Meteorology
Lesson 7 – Stability
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References and Reading: MT Chapter 7

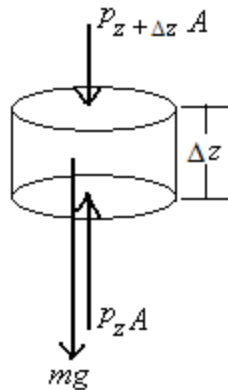
RELATION OF CLOUD FORMATION TO UPWARD MOTION

- As an air parcel is lifted (for whatever reason), it cools due to adiabatic expansion.
- Eventually it will cool enough to reach saturation, at which point condensation will occur and a cloud will form.
- Most clouds in the atmosphere form as a result of upward vertical motion.
- Upward motion caused by four main lifting mechanisms
 - Convective lifting
 - Orographic lifting
 - Convergence
 - Frontal lifting

BUOYANCY

- We will use a prime notation to indicate properties of the air parcel, and non-primed quantities are properties of the surrounding environment.
- The vertical force balance on an air parcel of volume $V = A\Delta z$ (see diagram) is

$$\sum F_z = -p_{z+\Delta z}A + p_zA - \rho'Vg . \quad (1)$$



which recognizing that $A = V/\Delta z$ rearranges to

$$\sum F_z = -\frac{p_{z+\Delta z} - p_z}{\Delta z}V - \rho'gV$$

or, as $\Delta z \rightarrow 0$,

$$\sum F_z = -\frac{dp}{dz}V - \rho'gV. \quad (2)$$

- We can use the hydrostatic equation for the environmental air, so that

$$\frac{dp}{dz} = -\rho g$$

so the sum of the forces becomes

$$\sum F_z = \rho gV - \rho'gV. \quad (3)$$

- Equation (3) shows that there are two forces acting on the air parcel...a downward force due to the weight of the air parcel itself ($\rho'Vg$), and an upward force equal to the weight of the environmental air displaced by the air parcel (ρVg).
 - This is Archimedes's principle...there is a buoyant force on an object equal to the weight of any fluid it displaces.
 - This is the reason why ships and balloons can float.
- From Newton's second law we know that $\sum F_z = m'a_z = \rho'Va_z$. Using this in (3) leads to an equation for the vertical acceleration of an air parcel

$$a_z = \frac{(\rho - \rho')}{\rho'} g. \quad (4)$$

- If the parcel is denser than the environment, the acceleration will be downward. If it is lighter than the environment, the acceleration will be upward.
- Substituting for density from the ideal gas law, and assuming the pressure of the air parcel is the same as the pressure of the environment ($p = p'$), we can write the acceleration in terms of temperature

$$a_z = \frac{(T' - T)}{T} g. \quad (5)$$

- This shows us that warm air rises and cold air sinks.

DRY ADIABATIC LAPSE RATE

- An unsaturated air parcel that rises will cool at a rate given by

$$\frac{dT'}{dz} = -\frac{g}{c_p}. \quad (6)$$

- This formula says that if you lift an air parcel adiabatically, its temperature will decrease, which makes physical sense because the parcel will be expanding.

- We therefore can define a *dry adiabatic lapse rate* as

$$\Gamma_d \equiv -\left(\frac{dT'}{dz}\right)_{adiabatic} = \frac{g}{c_p}. \text{ Dry Adiabatic Lapse Rate} \quad (7)$$

- For dry air $c_p = 1007 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$, so that $\Gamma_d = 9.8^\circ\text{C}/\text{km}$.

STABILITY IN A DRY ATMOSPHERE

- Stability refers to whether an air parcel, one moved vertically, will continue to accelerate in the direction that it was pushed (unstable), or return in the direction from which it came (stable).
- We've already established that to determine the acceleration on the air parcel we need to compare its temperature with that of its surroundings.
- Imagine an air parcel that is in equilibrium with the environment, so that $T' = T = T_0$. There will be no acceleration of the air parcel, so the air parcel will remain at rest.
 - If the air parcel is initially at the origin, and is displaced a distance z , T' will change according to the adiabatic lapse rate so that

$$T'(z) = T_0 - \Gamma_d z. \quad (8)$$

- At altitude z , the environmental temperature is

$$T(z) = T_0 - \gamma z, \quad (9)$$

where γ is the environmental lapse rate.

- Substituting (8) and (9) into (5) shows that

$$a_z = \frac{\gamma - \Gamma_d}{T_0 - \gamma z} g z. \quad (10)$$

- Whether the parcel has an upward, downward, or no acceleration depends on how the environmental lapse rate compares with the dry-adiabatic lapse rate.
 - If $\Gamma_d < \gamma$, then the parcel will continue to accelerate upward after it was displaced upward, and therefore the atmosphere is unstable.
 - $\Gamma_d = \gamma$, then the parcel will remain where it is after it was displaced upward, and therefore the atmosphere is neutral
 - $\Gamma_d > \gamma$, then the parcel will accelerate downward after it was displaced upward, and therefore the atmosphere is stable.

POTENTIAL TEMPERATURE AND STABILITY

- Potential temperature is defined as the temperature an air parcel would have if it were moved dry-adiabatically to a reference pressure $p_0 = 1000$ mb,

$$\theta = T \left(\frac{p_0}{p} \right)^{R_d/c_p} . \quad (11)$$

- *In an adiabatic process potential temperature is conserved.*
- The vertical acceleration on an air parcel can be written in terms of potential temperature,

$$a_z = \frac{(\theta' - \theta)}{\theta} g . \quad (12)$$

- Imagine an air parcel that is in equilibrium with the environment, so that $\theta' = \theta = \theta_0$. There will be no acceleration of the air parcel, so the air parcel will remain at rest.
 - If the air parcel is initially at the origin, and is displaced a distance z , θ' will remain constant (since this is an adiabatic process).
 - At altitude z , the environmental potential temperature is

$$\theta(z) = \theta_0 + \frac{d\theta}{dz} z .$$

- The acceleration at altitude z is [from Eqn. (12)]

$$a_z = \frac{-d\theta/dz}{\theta(z)} g z . \quad (13)$$

- Thus, whether the parcel has an upward, downward, or no acceleration depends on how the environmental potential temperature changes with height.

$$\begin{aligned}\frac{d\theta}{dz} &> 0; \quad \text{stable} \\ \frac{d\theta}{dz} &= 0; \quad \text{neutral} \\ \frac{d\theta}{dz} &< 0; \quad \text{unstable}\end{aligned}$$

STABILITY IN A MOIST ATMOSPHERE

- The presence of water vapor changes our stability determination somewhat.
- Unless the air parcel is saturated, we can treat it as though it is completely dry for stability purposes.
- Once an air parcel is saturated, any further cooling will result in the release of latent heat due to condensation.
 - The latent heat released will result in the temperature of the air parcel cooling at lesser rate as it rises than it would if it were dry.
- The lapse rate at which a saturated air parcel cools if lifted is called the *saturated-adiabatic lapse rate*, and is a complicated function of temperature,

$$\Gamma_s = \Gamma_d \left(\frac{1 + \frac{r_s L_v}{R_d T}}{1 + \frac{L_v^2 r_s \epsilon}{c_p R_d T^2}} \right). \quad (14)$$

- The saturated-adiabatic lapse rate is always less than the dry adiabatic lapse rate.
 - In very humid conditions near the ground, Γ_s is around 4°C/km.
 - At very cold temperatures, Γ_s approaches Γ_d (because the air is so dry that there is little latent heat release).
- To assess the stability of a moist atmosphere, we must compare the environmental lapse rate with both the dry- and saturated-adiabatic lapse rates.
 - $\Gamma_d < \gamma$ and $\Gamma_s < \gamma$: absolutely unstable
 - $\Gamma_d > \gamma$ and $\Gamma_s < \gamma$: conditionally unstable
 - $\Gamma_d > \gamma$ and $\Gamma_s > \gamma$: absolutely stable

CHANGES TO STABILITY

- **Upward motion of a layer of air causes the layer to become less stable.**
- **Downward motion (subsidence) of a layer of air causes the layer to become more stable.**
- **Heating from below or cooling from above causes a layer of air to become less stable.**
- **Cooling from below or heating from above causes a layer of air to become more stable.**
- **Adding moisture at the lower bounds of a layer will cause the layer to become less stable.**

EXERCISES

1. A dry air parcel has a temperature of 20°C . The environmental lapse rate is $5^{\circ}\text{C}/\text{km}$. The air parcel is forced to rise over a mountain that is 3 km high.
 - a. What is the temperature of the air parcel at the top of the mountain?
 - b. What is the temperature of the environment at the top of the mountain?
 - c. What is the buoyant acceleration of the air parcel?
 - d. Is the atmosphere stable or unstable?
2. A moist air parcel has a temperature of 20°C , and is forced to rise over the same mountain as in problem 1. If it reaches saturation while it is ascending, will it be warmer or colder than the dry air parcel when it reaches the top of the mountain?
3. The dry and moist air parcels from problems 1 and 2 now are forced to descend the other side of the mountain. They both descend dry adiabatically. Will their temperatures be the same once they reach the bottom? If not, which one will be warmer?
4. For the following data, find the potential temperature at the two altitudes. Is the atmosphere stable or unstable?

Altitude (m)	Pressure (mb)	Temp ($^{\circ}\text{C}$)	θ (K)
1480	850	7	
5700	500	-15	

5. Show that

$$\frac{\rho - \rho'}{\rho'} = \frac{T' - T}{T} = \frac{\theta' - \theta}{\theta}.$$

What assumptions did you have to make? Are they reasonable?

6. Start with the ideal gas law and differentiate it with respect to z . Show that if the lapse rate is greater than g/R_d then density will increase with height, $d\rho/dz > 0$. This lapse rate is known as the *autoconvective* lapse rate, and is the reason that dust devils form.