

**ESCI 241 – Meteorology
Lesson 0 – Math and Physics Review**

UNITS

- A number is meaningless unless it is accompanied by a unit telling what the number represents.
- The standard unit system used internationally by scientists is known as the SI unit system. The basic units needed for a system of units are length, mass, and time. In the SI system, these are the meter (m), kilogram (kg), and second (s). Nearly every other unit can be derived from these three basic units. The SI unit system is sometimes referred to as the *m-k-s unit system* (as opposed to the c-g-s system, which uses centimeters, grams, and seconds as the basic units).
- Important units to remember are:

Phenomenon	Unit name	Basic units	Alternate units (non-SI)
Force	Newton (N)	kg-m-s ⁻²	dyne; pound
Energy	Joule (J)	N-m	erg*; foot-lb; calorie
Power	Watt (W)	J-s ⁻¹	Horsepower
Pressure	Pascal (Pa)	N-m ⁻²	lb-in ⁻² ; bar; torr; atmosphere; in-Hg
Temperature	Kelvin (K)	none	Celsius; Fahrenheit

- Prefixes for units:

Multiplier	Name	Abb.
10 ⁹	giga	G
10 ⁶	mega	M
10 ³	kilo	k
10 ⁻³	milli	m
10 ⁻⁶	micro	μ
10 ⁻⁹	nano	n

- Though internationally meteorologists adhere to SI units, in the U.S. we continue to use some traditional units that differ from SI units. Some of these are
 - Pressure: millibar (mb) = 10² Pa = hecta-Pascal (hPa)

atmosphere (atm) = 101325 Pa = 1013.25 mb

inches of mercury (in-Hg) – 29.92 in-Hg = 1013.25 mb = 1 atm

o Temperature: Celcius ($^{\circ}\text{C}$) = $\text{K} - 273.15$

Fahrenheit ($^{\circ}\text{F}$) = $(9/5)^{\circ}\text{C} + 32$

o Length: statute mile (mi) = 1.61 km = 1760 yds

nautical mile (M) = 1.15 mi = 2000 yds

o Speed: 1 Knot (kt) = 1 nautical mile per hour = 1.15 mph $\approx 0.5 \text{ m}\cdot\text{s}^{-1}$

o Energy: Calorie (cal) = 4.184 J

CONVERTING UNITS

- When multiplying or dividing numbers, you must keep track of units.
 - o Treat units just like they were numbers or variables.
 - o Units can cancel if they appear to the same power on both the top and the bottom of a fraction.
 - o You can multiply or divide both sides of an equation by a unit to try to cancel them.
 - o *Both sides of an equation must have the same units, or something is wrong!*
 - o Units must make sense physically

Examples of Unit Conversion

● Convert 25 cm to meters: $25 \text{ cm} \times \frac{1 \text{ m}}{10^2 \text{ cm}} = 0.25 \text{ m}$

● Convert 25 cm to μm : $25 \text{ cm} \times \frac{1 \text{ m}}{10^2 \text{ cm}} \times \frac{10^6 \mu\text{m}}{1 \text{ m}} = 2.5 \times 10^5 \mu\text{m}$

● Convert 14.66 lb-in⁻² to mb:

$$14.66 \frac{\text{lb}}{\text{in}^2} \times \frac{4.46 \text{ N}}{1 \text{ lb}} \times \left(\frac{39.37 \text{ in}}{\text{m}} \right)^2 = 1.013 \times 10^5 \frac{\text{N}}{\text{m}^2} = 1.013 \times 10^5 \text{ Pa} \times \frac{10^{-2} \text{ hPa}}{\text{Pa}} = 1.013 \times 10^3 \text{ hPa}$$

$$1.013 \times 10^3 \text{ hPa} \times \frac{1 \text{ mb}}{1 \text{ hPa}} = 1.013 \times 10^3 \text{ mb} = 1013 \text{ mb}$$

CAUTION WHEN CONVERTING TEMPERATURE CHANGES

- Students often get confused when converting temperature *changes*, rather than temperatures themselves. This is illustrated by the following problem:
 - If the temperature is falling by 3°C per hour, what is the rate of temperature change in Kelvin per hour?
 - Many students will make the mistake of converting 3°C to 276.15 K using the formula $K = ^\circ C + 273.15$, and then giving the answer of 276.15 K/hr. **DON'T DO THIS!!!**
- When dealing with changes in temperature, rather than temperature itself, you don't convert temperature using the traditional formulas of
 - $K = ^\circ C + 273.15$
 - Fahrenheit ($^{\circ}F$) = $(9/5)^{\circ}C + 32$
- Instead, you have to realize that a *change* of 1°C is the exact same as a change of 1K, so a change of 3°C/hr = 3 K/hr.
- If you are converting a temperature change from Celsius to Fahrenheit, to need to recognize that a *change* of 1°C is the same as a change of 1.8°F.
- This often comes up when dealing with specific heats, lapse rates, or any place where it is the change in temperature that is important. You are now forewarned!

RATIOS OF TEMPERATURE VS RATIOS OF OTHER UNITS

- When we have ratios of pressures such as p_1/p_2 it doesn't matter what units for pressure we use as long as they are the same on the top and bottom.

$$\frac{500 \text{ mb}}{1000 \text{ mb}} = \frac{14.76 \text{ in Hg}}{29.53 \text{ in Hg}} = \frac{1}{2}$$

- But when we have ratios of temperature such as T_1/T_2 , **we must use Kelvin!**

$$\frac{373.15K}{273.15K} = 1.37$$
$$\frac{100^{\circ}C}{0^{\circ}C} = \infty$$

- The difference between the two is that the different pressure scales all have the same zero value, while the different temperature scales do not share the same zero values!

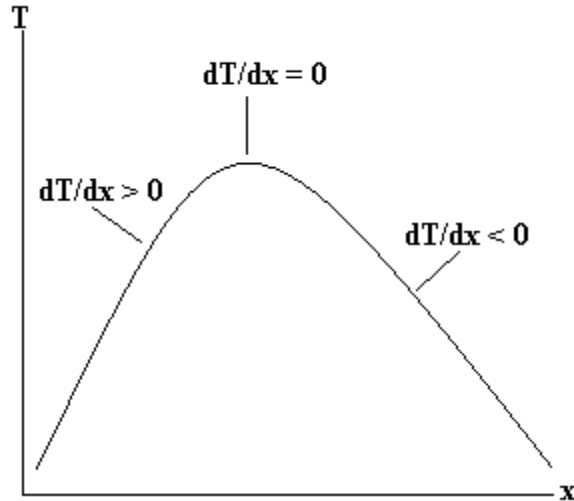
COORDINATES AND VELOCITY

- In meteorology we use the following coordinate system:
 - The x -coordinate increases eastward
 - The y -coordinate increases northward
 - The z -coordinate increases upward
- The velocity components along each coordinate direction are defined as
 - $u \equiv dx/dt$; u is the speed in the eastward direction (*zonal* velocity)
 - $v \equiv dy/dt$; v is the speed in the northward direction (*meridional* velocity)
 - $w \equiv dz/dt$; w is the speed in the upward direction (*vertical* velocity)

DERIVATIVES

- A derivative tells us how one variable changes due to changes in another variable.
 - For example, dT/dx refers to how temperature (T) changes as we move eastward (toward increasing x). If temperature increases toward the East then dT/dx is positive. If temperature decreases toward the East then dT/dx is negative.

- The derivative of a function is the slope of a tangent to the function. A positive slope means a positive derivative; negative slope means a negative derivative; zero slope means zero for the derivative (see picture below).



- We will often use partial derivative notation in this class, which means sometimes derivatives will be written as $\partial T/\partial x$ instead of dT/dx .
 - For those who haven't been exposed to partial derivatives, don't panic...we will use partial derivatives pretty much in the same way as normal derivatives. Just think of it as how T changes with x .
- Velocity and acceleration are related to the time derivatives of the coordinates.

Coordinate	Velocity	Acceleration
x	$u \equiv \frac{dx}{dt}$	$a_x \equiv \frac{d^2 x}{dt^2} = \frac{du}{dt}$
y	$v \equiv \frac{dy}{dt}$	$a_y \equiv \frac{d^2 y}{dt^2} = \frac{dv}{dt}$
z	$w \equiv \frac{dz}{dt}$	$a_z \equiv \frac{d^2 z}{dt^2} = \frac{dw}{dt}$

INTEGRALS

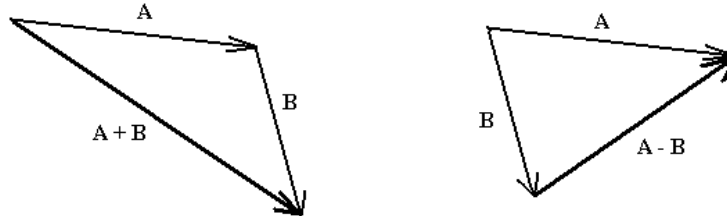
- An integral is an antiderivative.

$$\int \frac{df}{dx} dx = f(x) + const.$$

- An integral of a function represents the area under the function.

VECTORS

- A vector consists of a direction and a magnitude.
- Two vectors are added by placing them head to tail



- A vector can be written in terms of its components along each coordinate direction.

$$\vec{A} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{B} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

- When two vectors are added or subtracted, their components are added or subtracted.

$$\vec{A} + \vec{B} = (a_x + b_x) \hat{i} + (a_y + b_y) \hat{j} + (a_z + b_z) \hat{k}$$

$$\vec{A} - \vec{B} = (a_x - b_x) \hat{i} + (a_y - b_y) \hat{j} + (a_z - b_z) \hat{k}$$

- The magnitude of a vector can be found from its components.

$$|\vec{A}| \equiv \sqrt{a_x^2 + a_y^2 + a_z^2}$$

GRADIENT

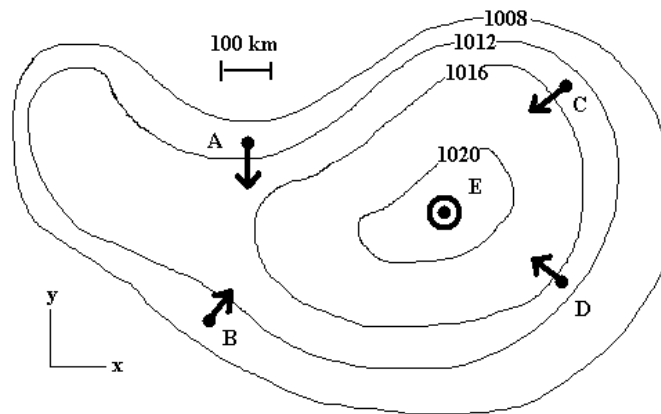
- The *gradient* (or *del*) operator is defined as

$$\nabla \equiv \left(\frac{\partial}{\partial x} \right) \hat{i} + \left(\frac{\partial}{\partial y} \right) \hat{j} + \left(\frac{\partial}{\partial z} \right) \hat{k}.$$

- The *gradient* of a scalar field (such as pressure) is a vector pointing in the direction of maximum increase in the field. It is defined as

$$\nabla p \equiv \left(\frac{\partial p}{\partial x} \right) \hat{i} + \left(\frac{\partial p}{\partial y} \right) \hat{j} + \left(\frac{\partial p}{\partial z} \right) \hat{k}.$$

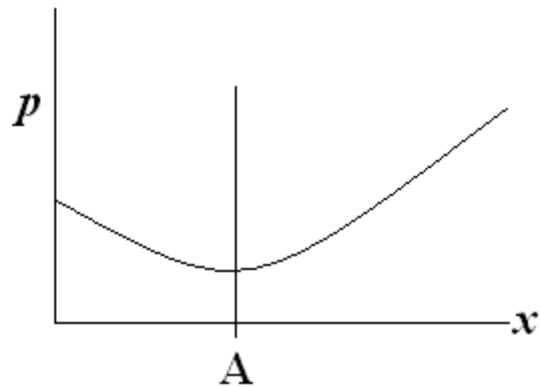
- If a contour of the scalar field is plotted (such as isotherms or isobars) the gradient at a given point is a vector that is oriented at 90° to the contours and pointed toward higher values.
 - The example below shows the direction of the pressure gradient at several points.



- Since the gradient is a vector, it has components in the x - and y -directions. The table below shows the sign of the components of the pressure gradient at the five points from the example above.

Point	$\partial p/\partial x$	$\partial p/\partial y$
A	0	-
B	+	+
C	-	-
D	-	+
E	0	0

- If it isn't apparent to you why at point A is zero, imagine that you are walking from west to east across point A and are carrying a barometer (or barograph). The pressure trace would look something like the following figure.

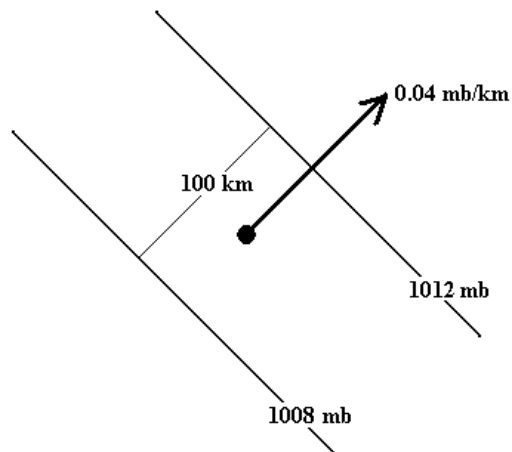


The slope of the pressure trace is zero at point A, and therefore $\partial p / \partial x = 0$ at point A.

- The magnitude of the gradient at a point on a contour map can be found by dividing the contour interval by the shortest distance between contours across the point. In the example below the magnitude and direction of the pressure gradient is shown.

- The magnitude was found by

$$|\nabla p| \cong \frac{1012\text{mb} - 1008\text{mb}}{100\text{km}} = 0.04\text{mb} / \text{km}$$



- The magnitude of the gradient increases if the contours become closer together, and decreases as they get farther apart.
- When the contours are packed closely together it is often referred to as a *tight gradient*.

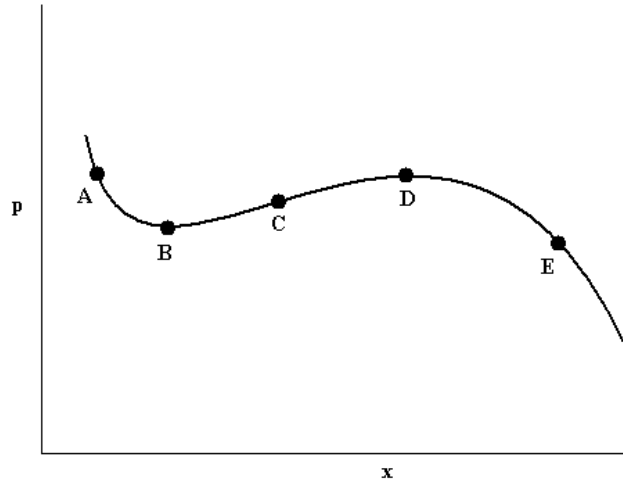
- When the contours are far apart it is often referred to as a *loose gradient*.

READING EQUATIONS

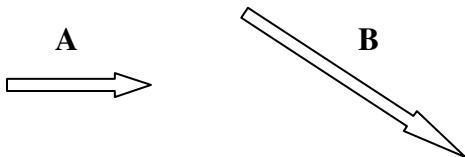
- Meteorology is really a branch of applied physics, and the language of physics is mathematics.
- Our understanding of the atmosphere could not have occurred without mathematics as a foundation.
- Mathematics and equations are really just another language. If you attempted to write down the advanced concepts of meteorology without using mathematics or equations, you would quickly run out of paper and time. Equations are a shorthand way of expressing physical concepts.
- *The key to using math and equations properly in the study of meteorology is to learn how to read equations.*
 - The way you learn to read them is to practice (just like if you were learning another language).
 - *Don't immediately try to plug numbers into an equation!* Instead, try to figure out what the equation is saying about the physical world. Often we will use and write down equations without ever using them to find an actual numerical value.

EXERCISES

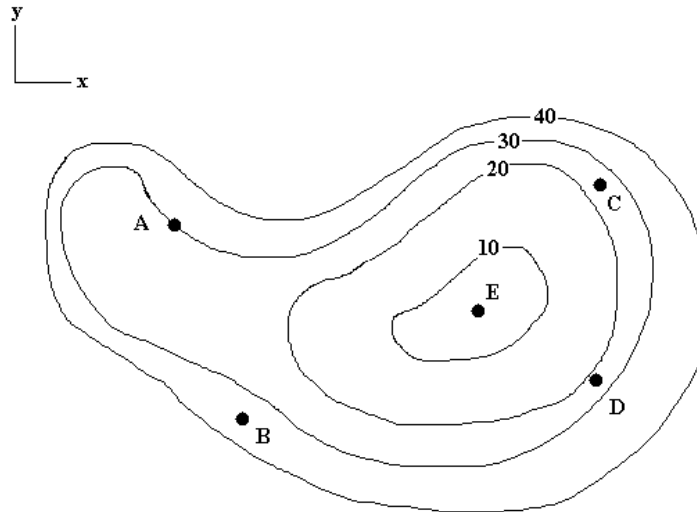
1. For the diagram below, tell whether $\frac{dp}{dx}$ is positive, negative, or zero at each point A, B, C, D, and E.



2. For the vector pair (A and B) shown below, draw additional vectors that represent $A + B$ and $A - B$.



3. a. The diagram below represents temperatures in °F. At each point A, B, C, D, and E draw an arrow indicating the direction of the gradient of temperature, ∇T . (Note that E lies at the lowest temperature).



- b. Fill in the table below telling whether $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$ are positive, negative, or zero at each point.

Point	$\frac{\partial T}{\partial x}$	$\frac{\partial T}{\partial y}$
A		
B		
C		
D		
E		

- c. Which point has the strongest temperature gradient?
- d. Which point has the weakest temperature gradient?

4. For the following isotherm pattern, find the magnitude and direction of the temperature gradient at the two points A and B.

