**Virtual Temperature Approximation Using Celsius Temperatures and Dimensional Mixing Ratio**

Virtual temperature is defined as

$$T_v = T\left(1+0.61q\right) \cong T\left(1+0.61r\right) \tag{1}$$

where *T* is absolute temperature, and *q* and *r* are the dimensionless specific humidity and mixing ratio respectively. A common approximation for virtual temperature is

$$T_v \cong T + \frac{r}{6} \tag{2}$$

where the temperatures are in °C and the mixing ratio is in g/kg. The validity of this approximation is demonstrated with the following proof.

Defining $T'$ to be the Celsius temperature, $T_v'$ to be the Celsius virtual temperature, and $r'$ to be the dimensional mixing ratio (g/kg), the conversion factors are

$$\begin{aligned} T_v &= T_v' + 273 \\ T &= T' + 273 \\ r' &= 1000r\,. \end{aligned} \tag{3}$$

Using these in Eq. (1) yields,

$$273 + T_v' \cong \left(273 + T'\right)\left(1 + \frac{0.61r'}{1000}\right)$$

which expands as

$$273 + T_v' \cong 273 + 0.167r' + T' + 6.1\times10^{-4}T'r'$$

or

$$T_v' \cong 0.167r' + T' + 6.1\times10^{-4}T'r'$$

or finally

$$T_v' \cong T'\left(1 + 6.1\times10^{-4}r'\right) + 0.167r'\,. \tag{4}$$

The largest mixing ratios encountered in the atmosphere are of the order of 40 g/kg, so the largest the term $6.1\times10^{-4}r'$ will be is about 0.025. We can therefore ignore this term in Eq. (4), so we are left with

$$T_v' \cong T' + 0.167r'\,.$$

Finally, 0.167 is approximately equal to 1/6, so we have

$$T_v' \cong T' + \frac{r'}{6}\,.$$